Parameterized Verification

Keeping Crowds Safe

Javier Esparza

Technical University of Munich

„Program Verification? Why don´t you give up?“

Theorem (Alan Turing, 1936)

Program termination is undecidable.

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Every non-trivial property of programs is undecidable.

Theorem (Marvin Minsky, 1969)

Every non-trivial property of while-programs with two counter variables is undecidable.

Because ...

- Undecidability requires some source of „infinity“:
  - Variables with an infinite range
  - Dynamic data structures (lists, trees)
  - Unbounded recursion
- Concurrent systems
  - are difficult to get right, and
  - often have a finite state space.

Dijkstra´s Mutual Exclusion Algorithm

Solution of a Problem in Concurrent Programming Control

E. W. DIJKSTRA
Technological University, Eindhoven, The Netherlands

A number of mainly independent sequential-cyclic processes with restricted means of communication with each other can be made in such a way that at any moment one and only one of them is engaged in the "critical section" of its cycle.

CACM 8:9, 1965

Concurrent programs are often finite-state

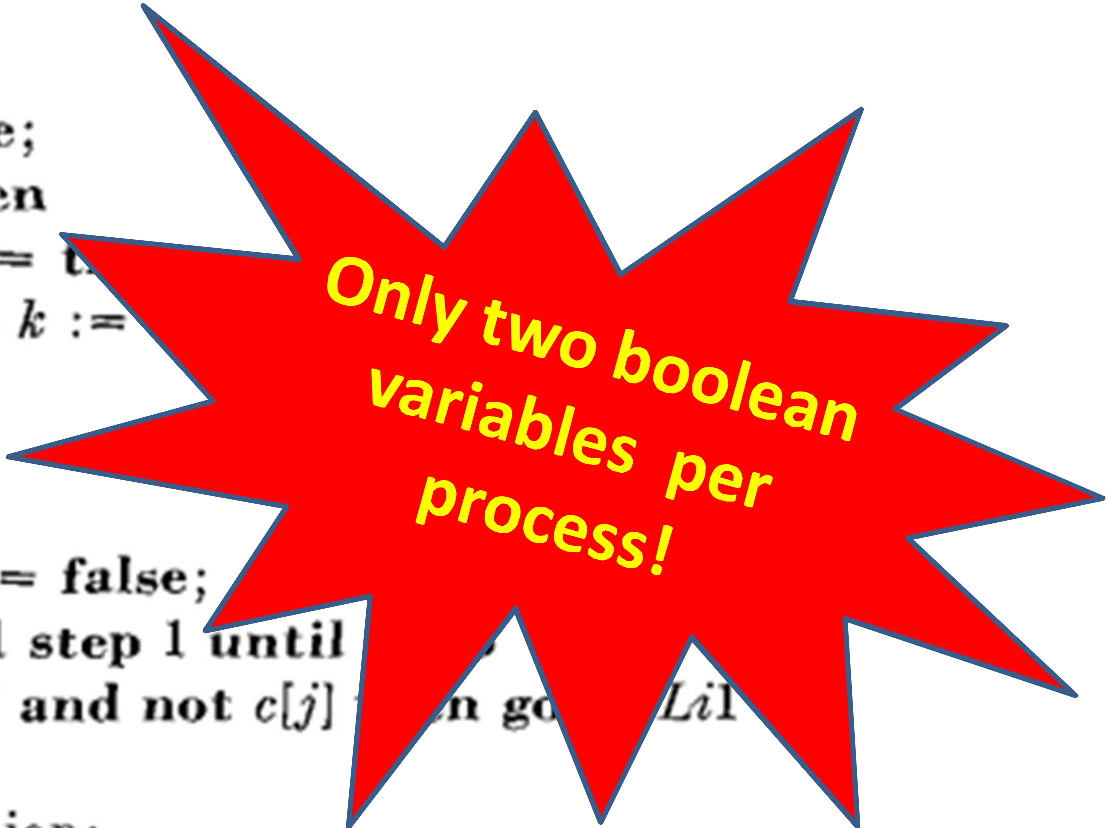
The program for the ith computer $(1 \le i \le N)$ is:

"**integer** j;
$Li0$: $b[i]$:= **false**;
$Li1$: **if** $k \neq i$ **then**
$Li2$: **begin** $c[i]$:= **true**;
$Li3$: **if** $b[k]$ **then** $k := i$;
go to $Li1$
end
else
$Li4$: **begin** $c[i]$:= **false**;
for j := 1 **step** 1 **until** N **do**
if $j \neq i$ **and not** $c[j]$ **then go to** $Li1$
end;
critical section;
$c[i]$:= **true**; $b[i]$:= **true**;
remainder of the cycle in which stopping is allowed;
go to $Li0$"

Concurrent programs are often finite-state

The program for the ith computer $(1 \le i \le N)$ is:

```
"integer j;
Li0:  b[i] := false;
Li1:  if k ≠ i then
Li2:  begin c[i] := t
Li3:  if b[k] then k :=
      go to Li1
      end
        else
Li4:  begin c[i] := false;
        for j := 1 step 1 until
          if j ≠ i and not c[j]      n go      Li1
      end;
      critical section;
      c[i] := true;  b[i] := true;
      remainder of the cycle in which stopping is allowed;
      go to Li0"
```



A Leader Election Algorithm (90s)

An $O(n \log n)$ Unidirectional Distributed Algorithm for Extrema Finding in a Circle

DANNY DOLEV, MARIA KLAWE, AND MICHAEL RODEH*

behavior of an active process v.

A0. Send the message $\langle 1, \max(v) \rangle$.

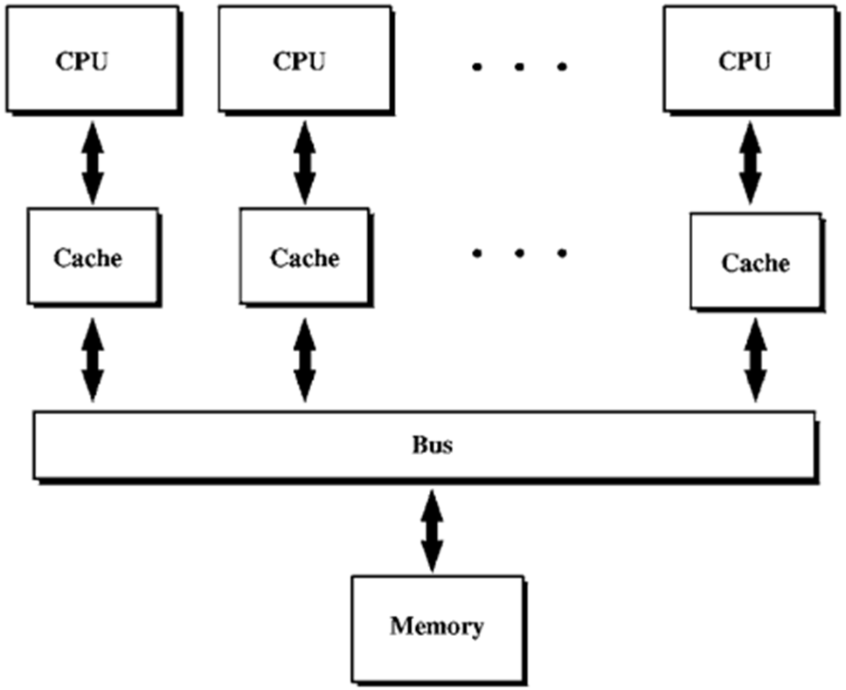
A1. If a message $\langle 1, i \rangle$ arrives do as follows:

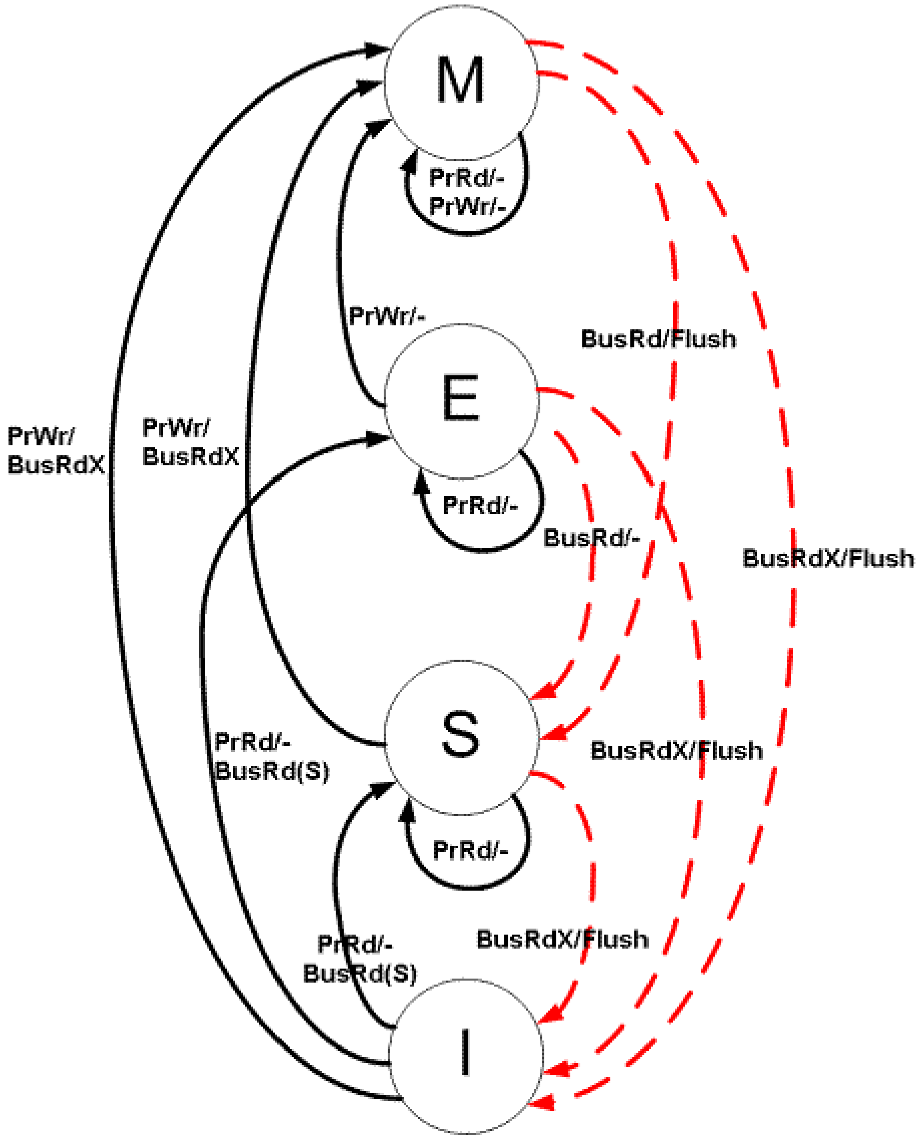
1. If $i \neq \max(v)$ then send the message $\langle 2, i \rangle$, and assign i to left(v).
2. Otherwise, halt—max(v) is the global maximum.

A2. If a message $\langle 2, j \rangle$ arrives do as follows:

1. If left(v) is greater than both j and max(v) then assign left (v) to max(v), and send the message $\langle 1, \max(v) \rangle$.
2. Otherwise, become passive.

A Cache-Coherence Protocol (00s)





Source: Wikipedia

A Model of a Bluetooth Driver (10s)

```
struct DEVICE_EXTENSION {
    int  pendingIo;
    bool stoppingFlag;
    bool stoppingEvent;
};

bool stopped;

void main() {
    DEVICE_EXTENSION *e =
        malloc(sizeof(DEVICE_EXTENSION));
    e->pendingIo = 1;
    e->stoppingFlag = false;
    e->stoppingEvent = false;
    stopped = false;
    async BCSP_PnpStop(e);
    BCSP_PnpAdd(e);
}
```

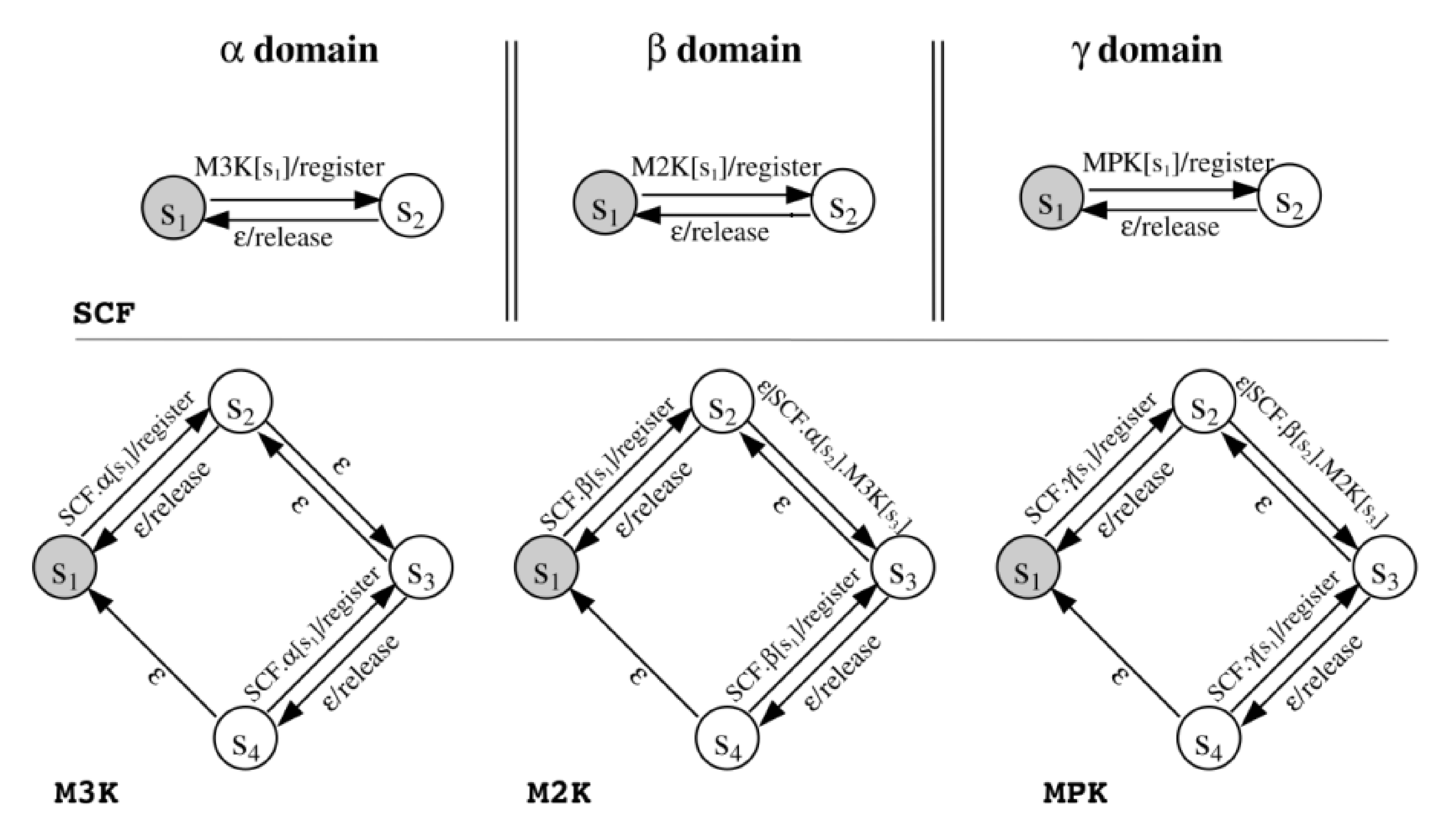
```
int BCSP_IoIncrement(DEVICE_EXTENSION *e) {
    if (e->stoppingFlag)
        return -1;
    atomic {
        e->pendingIo = e->pendingIo + 1;
    }
    return 0;
}

void BCSP_IoDecrement(DEVICE_EXTENSION *e) {
    int pendingIo;
    atomic {
        e->pendingIo = e->pendingIo - 1;
        pendingIo = e->pendingIo;
    }
    if (pendingIo == 0)
        e->stoppingEvent = true;
}
```

```
void BCSP_PnpAdd(DEVICE_EXTENSION *e) {
    int status;
    status = BCSP_IoIncrement (e);
    if (status == 0) {
        // do work here
        assert !stopped;
    }
    BCSP_IoDecrement(e);
}

void BCSP_PnpStop(DEVICE_EXTENSION *e) {
    e->stoppingFlag = true;
    BCSP_IoDecrement(e);
    assume e->stoppingEvent;
    // release allocated resources
    stopped = true;
}
```

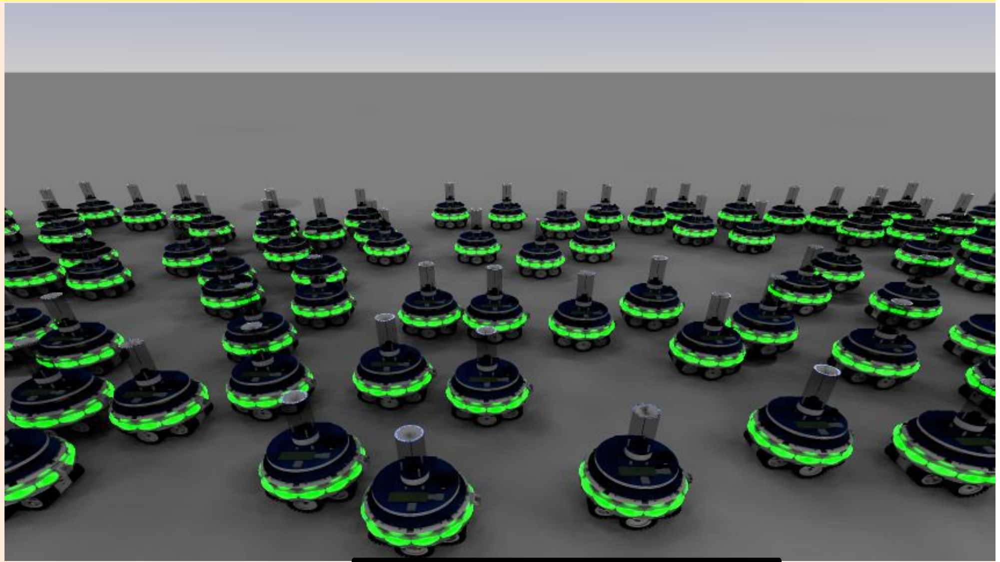
A Model of a Biochemical System (10s)



Source: Shangai Institutes for Biological Sciences

Robot swarms, flocks of birds, vehicular networks … (into the 20s?)

Source: Upenn



Source: Iridia-CoDE

Source: Wikipedia

Parameterized Verification

- Model-checking tools can only check instances of these systems for particular values of the number N of processes.

Can we prove correctness *for every N* ?

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Can we prove correctness *for every N* ?

- Amounts to checking an infinite family of finite-state systems.

Keeping a Crowd Safe

- The safety /coverability problem:
  - Given: a program template $T[i]$ with finite-range variables,

    a „dangerous“ control point ℓ of $T[i]$.
  - Decide: Is there a number N such that the crowd

    $$T[1] \parallel T[2] \parallel \cdots \parallel T[N]$$

    can reach a global state in which at least one of $T[1], T[2], \ldots, T[N]$ is at ℓ („covers“ ℓ) ?

Parameterized Verification: Give up?

Theorem (folklore): The Halting Problem can be reduced to the parameterized coverability problem.

Reduction:

– The template models the behaviour of one tape cell.

TM terminates

→ it uses a finite number N of cells

→ N copies of the template reach the dangerous control point

Parameterized Verification: Give up?

Reduction:

– The template models the behaviour of one tape cell.

global var: $state$: $\{q_0, \ldots, q_n\}$

var: $cell_i$: $\{a_1, \ldots a_m\}$
var: $active_i$: $\mathbf{boolean}$

if $active_i$ **then**

  ...
  if $state = q$ **and** $cell_i = a$ **then**
    $state \coloneqq q'$; $cell_i \coloneqq a'$; $(q, a) \rightarrow (q', a', R)$
    $active_i \coloneqq \mathbf{false}$; $active_{i+1} \coloneqq \mathbf{true}$;
  endif
  ...
endif

Parameterized Verification: Give up?

Reduction:

– The template models the behaviour of one tape cell.

global var: $state$: $\{q_0, \dots, q_n\}$

Hey, what happens at the right border?

...

if $state = q$ **and** $cell_i = a$ **then** $\qquad (q, a) \rightarrow (q', a', R)$

$state \coloneqq q'$; $cell_i \coloneqq a'$;

$active_i \coloneqq \mathbf{false}$; $active_{i+1} \coloneqq \mathbf{true}$;

endif

...

endif

Parameterized Verification: Give up?

Reduction:

– The template models the behaviour of one tape cell.

global var: $state$: $\{q_0, \dots, q_n\}$

Hey, what happens at the right border?

OK, just substitute $i+1 \; mod \; N$ for $i+1$!

...

if $state = q$ **and** ce[...] $= a$ **then**

$state \coloneqq q'; cell_i \coloneqq a$[...]$;$ $\quad (q, a) \rightarrow (q', a', R)$

$active_i \coloneqq \mathbf{false}; active_{i+1} \coloneqq \mathbf{true};$

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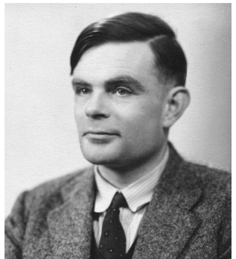

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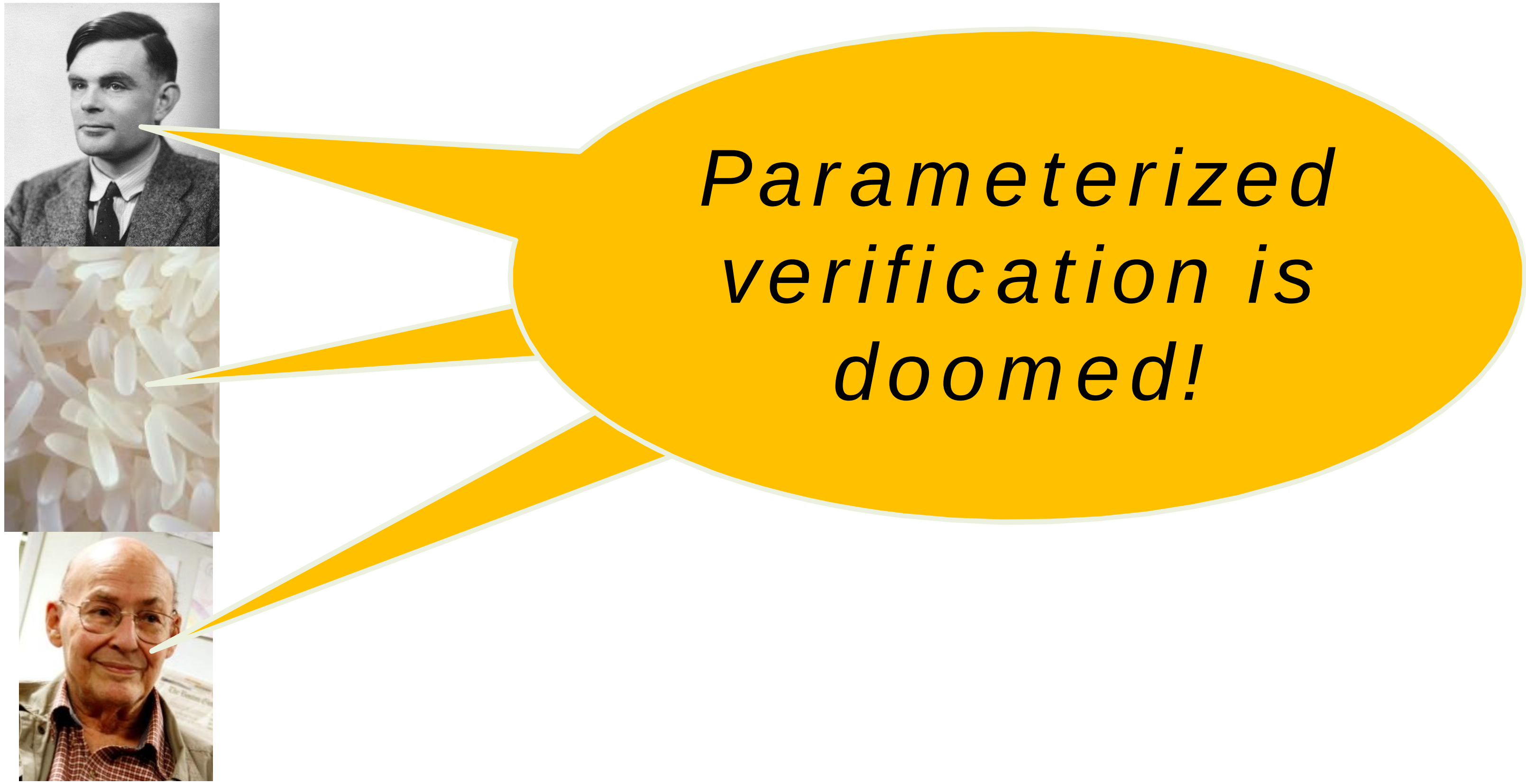

Identities

- In this reduction, processes do not execute exactly the same code
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- But many systems do not use identities:
  - DKR Leader Election uses identities.
  - Dijkstra, MESI-protocol, Bluetooth driver, biochemical systems do not.
- In other systems, processes must remain anonymous!

Anonymous Crowds

- Goal: investigate the decidability and complexity of the coverability problem for crowds in which
  (1) every process executes exactly the same code, (anonymous crowds), and
  (2) the number of processes is unknown to the processes.

Coverability Problem for Anonymous Crowds

- Given: A finite automaton) A (a template) and a „dangerous“ state q_d of A
- Decide: Is there a number N such that the anonymous crowd consisting of N copies of A can reach a global state that covers (puts at least one process in) state q_d?

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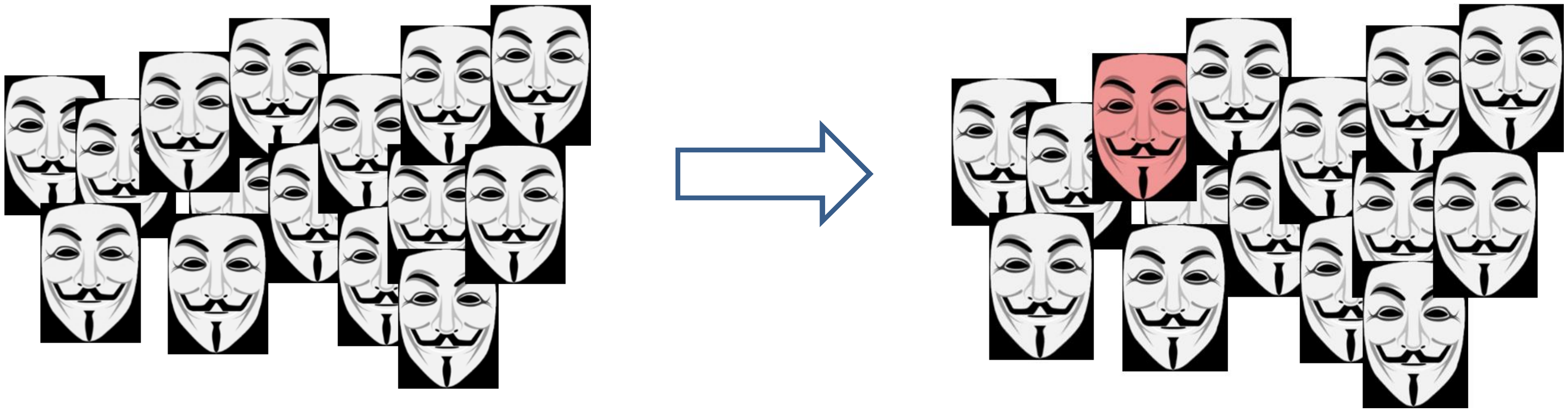

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- Initial configurations not yet specified.
- Communication mechanism not yet specified

Initial Configurations

- A configuration of the system is completely determined by the number of processes in each state of the template (no identities)
- Configuration: function $Q \to \mathbb{N}$, or vector of natural numbers with $|Q|$ components, or parallel expression

$$Q = \{q_1, q_2, q_3, q_4\}$$

(3, 0, 2, 1) $\qquad q_1^3 \parallel q_3^2 \parallel q_4$

Initial Sets of Configurations

- Parametrized configurations with leaders:
  - Exactly one process in some states
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- Two variants of the problem, depending on the initial parametrized configurations allowed
  - Crowds with leaders as DEFAULT option

Coverability Problem with Leaders

- Given: A template A,

  an initial parametrized configuration $\boldsymbol{\mathcal{C}}$ of A with leaders,

  a „dangerous" state q_d of A

- Decide: Is there a configuration $c \in \boldsymbol{\mathcal{C}}$ such that the crowd starting at c can reach a configuration covering q_d ?

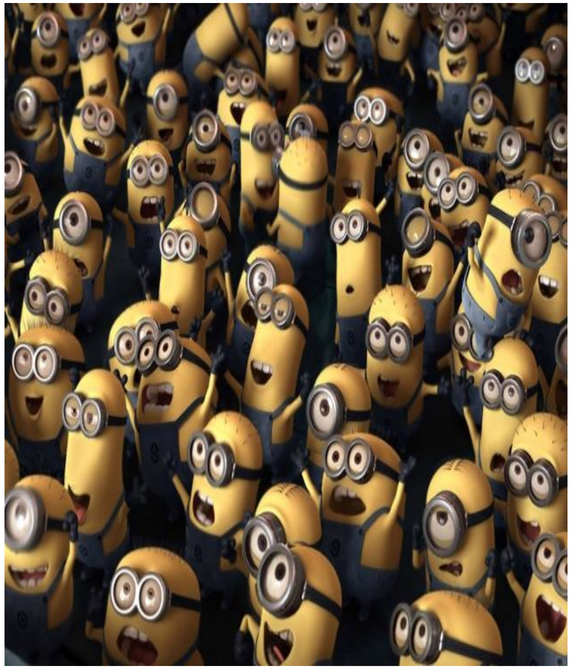
Equivalent Formulation

- Given: A finite set $A_1, \ldots, A_n$ of „leader templates“ with initial states $q_1, \ldots, q_n$ a finite set $B_1, \ldots, B_n$ of „crowd templates“ with initial states $r_1, \ldots, r_m$, a „dangerous“ state q_d
- Decide: Are there numbers $k_1, \ldots, k_m$ such that the configuration with
  - one process in each of $q_1, \ldots, q_n$
  - $k_1, \ldots, k_m$ processes in $r_1, \ldots, r_m$

  can reach a configuration covering q_d?

Coverability Problem without Leaders

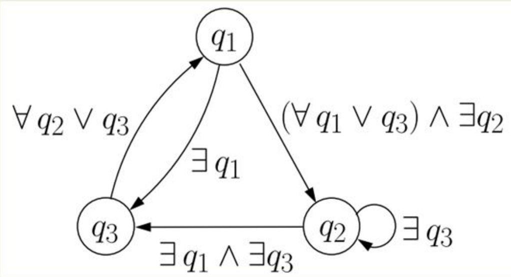
- Given: A template A,
  an initial parametrized leaderless configuration $\mathcal{C}$ of A
- Decide: Is there a configuration c $\in$ $\mathcal{C}$ such that the crowd starting at c can reach a configuration covering q_d ?



Zoo of Communication Mechanisms

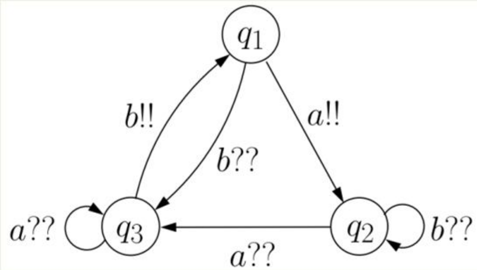
Global guards

State changes of a process depend on state of all others



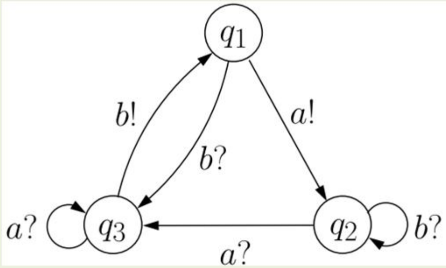
Broadcast

One process sends, everyone receives immediately



Rendezvous

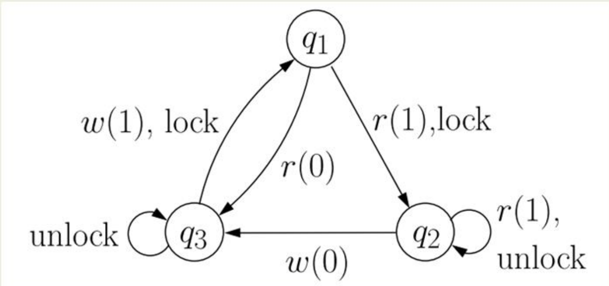
Synchronous exchange between two processes



Shared memory

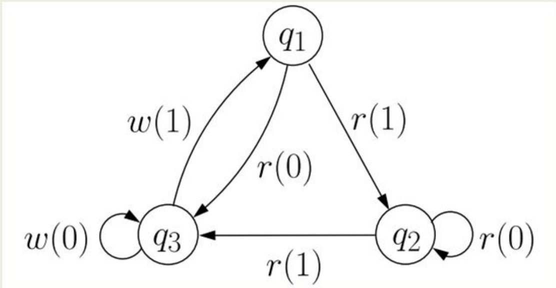
Processes compete for lock

Lock holder reads/writes shared state



Lock-free shared memory

No locks, interleaved reads/writes



High or Low Complexity?

Verifiers want low complexity

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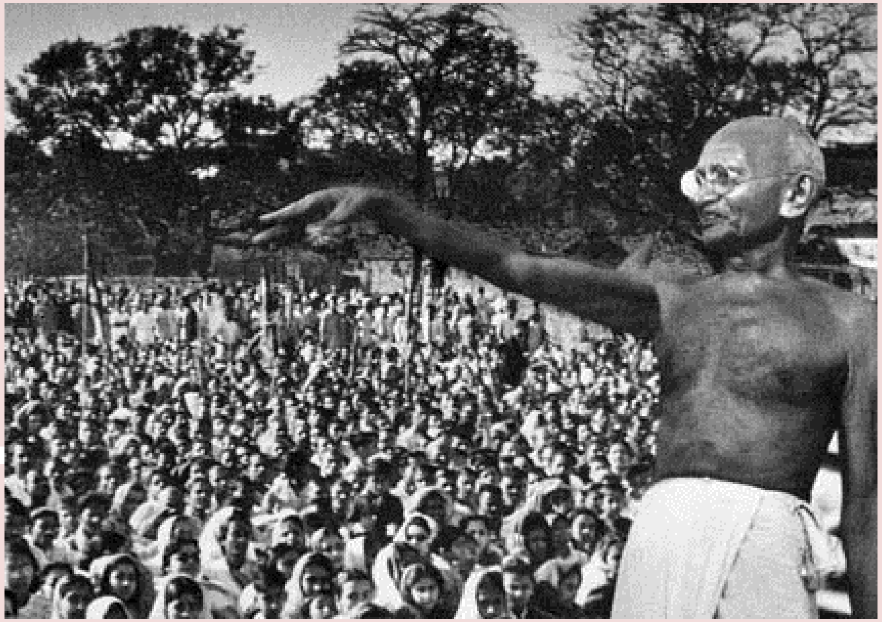
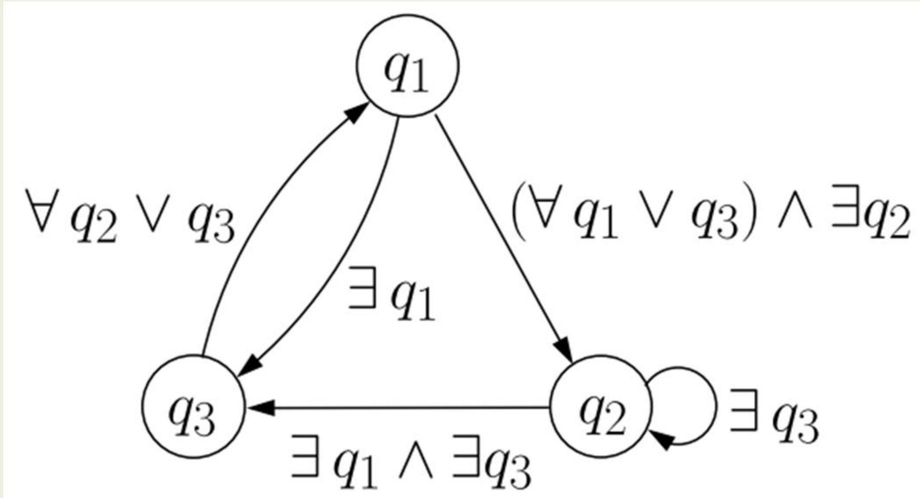


„Crowd designers“ (swarm intelligence, population protocols, crowdsourcing) want high complexity

Comm. Mechanisms: Global Guards

Global guards

- Process can make a move if the current state of all other processes satisfies some condition



Examples

- Abstractions of distributed algorithms (bakery, mutex)
- Very hard to implement

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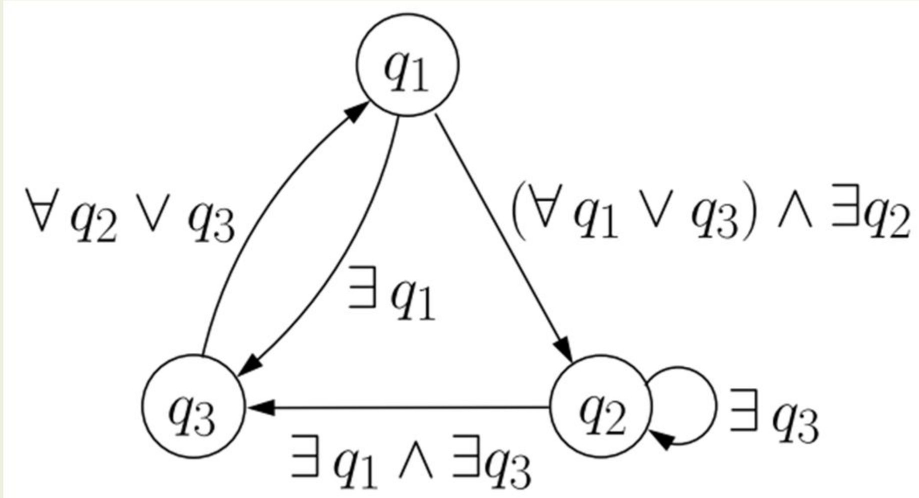


- Theorem (Emerson and Kahlon, LICS'03)
  The symmetric coverability problem is undecidable for systems communicating with global guards.

Counter Programs

- Sequence of labelled commands of the form
  - $\ell: x \coloneqq x + 1$
  - $\ell: x \coloneqq x - 1$
  - ℓ: $\mathbf{goto}\ \ell'$
  - ℓ: **if** $x = 0$ **then goto** ℓ' **else goto** ℓ''
  - ℓ: **halt**

Theorem (Minsky 69): The problem whether a counter program with all counters initialized to 0 halts is undecidable.

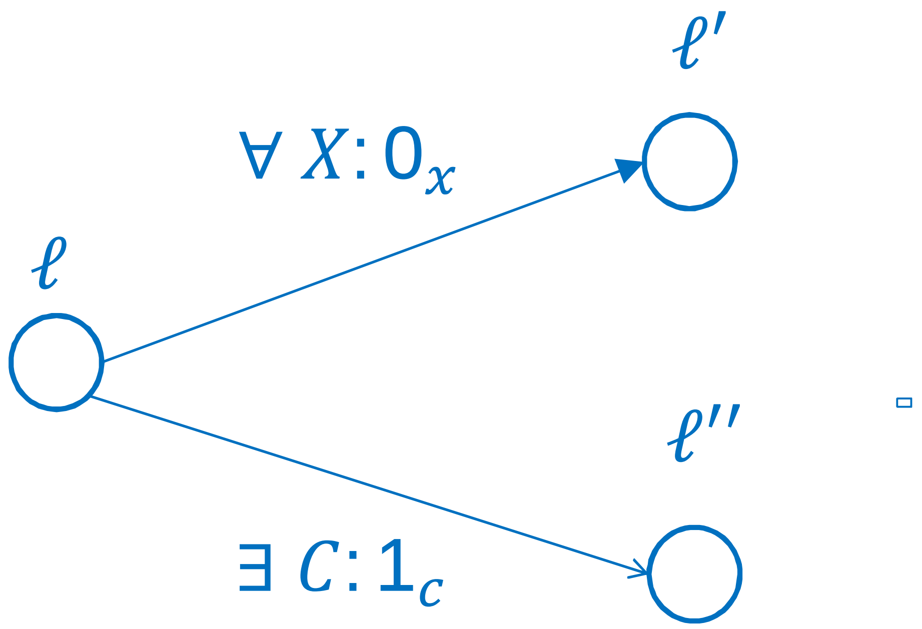
Simulating Counter Programs

- One control template T modelling the control flow of the program, with a state for each program label.
- One counter template X for each counter x, with two states: 0_x , 1_x

  Idea: „x has value n" modeled by n copies of X in state 1_x
- A configuration $(\ell, n_1, \dots, n_k)$ of the program is simulated by a configuration of the crowd with
  - one process in q_ℓ,
  - n_1 processes in 1_{x_1}, ..., n_k processes in 1_{x_k}
  - All other processes in 0-states

Simulating Counter Programs

- Simulating ℓ: if $x = 0$ then goto ℓ' else goto ℓ''



Simulating Counter Programs

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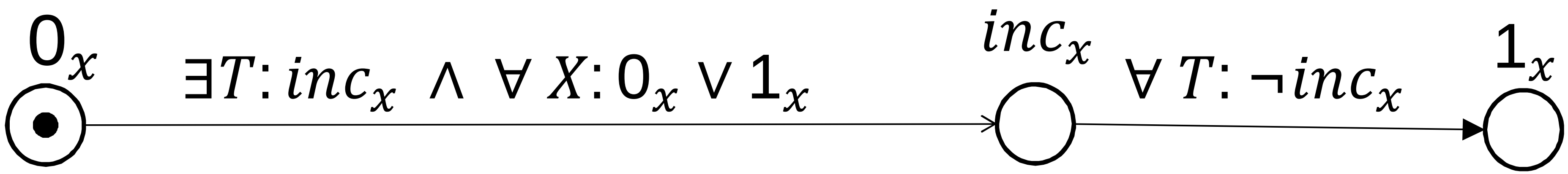

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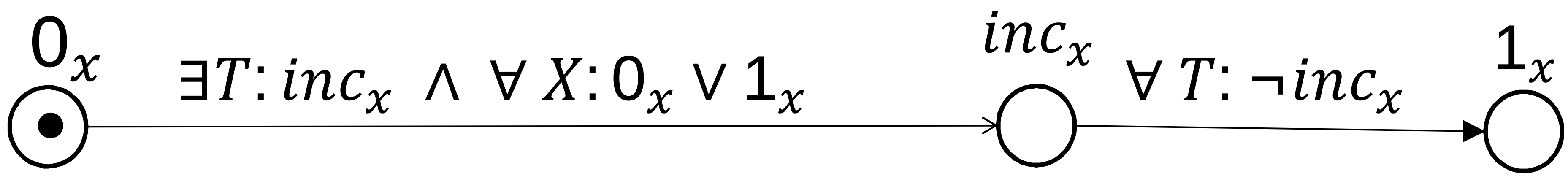

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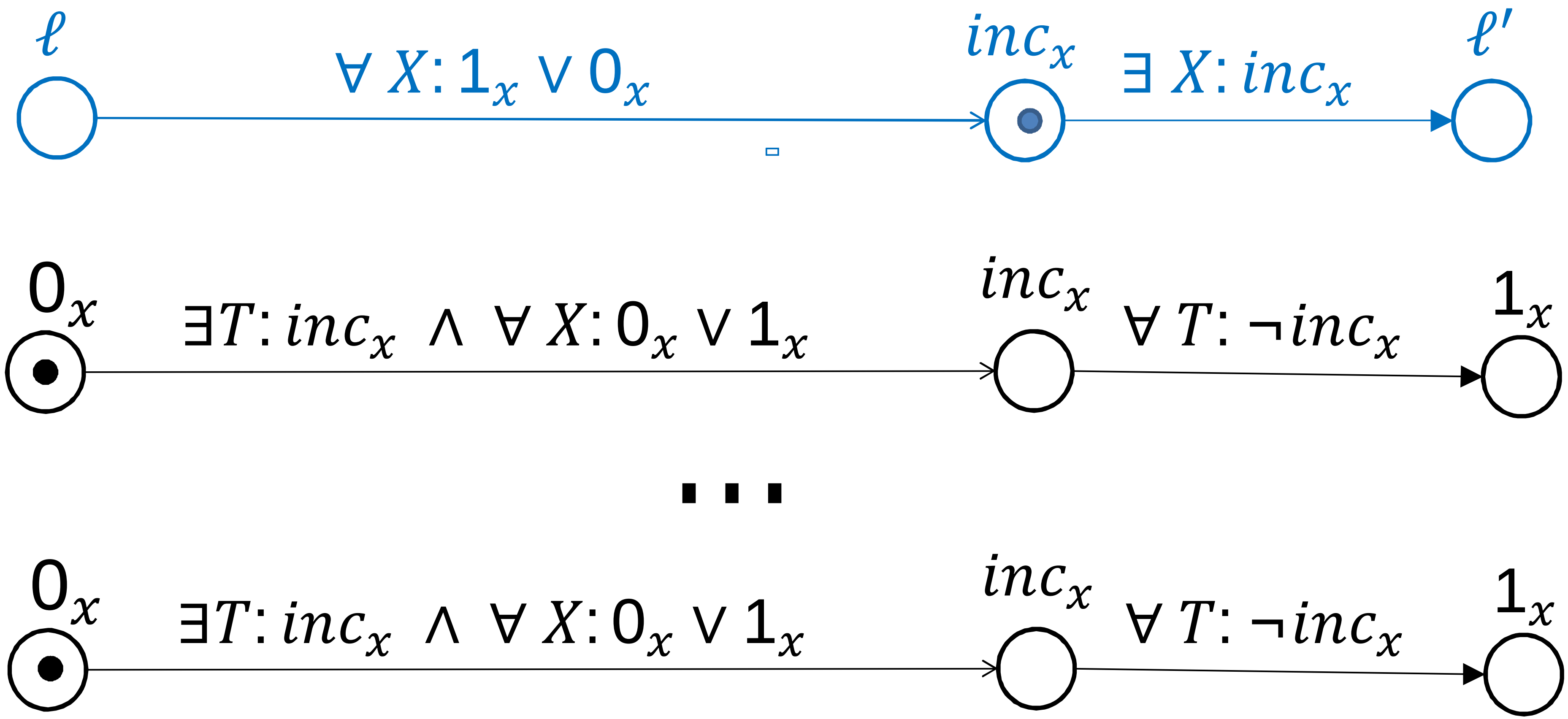

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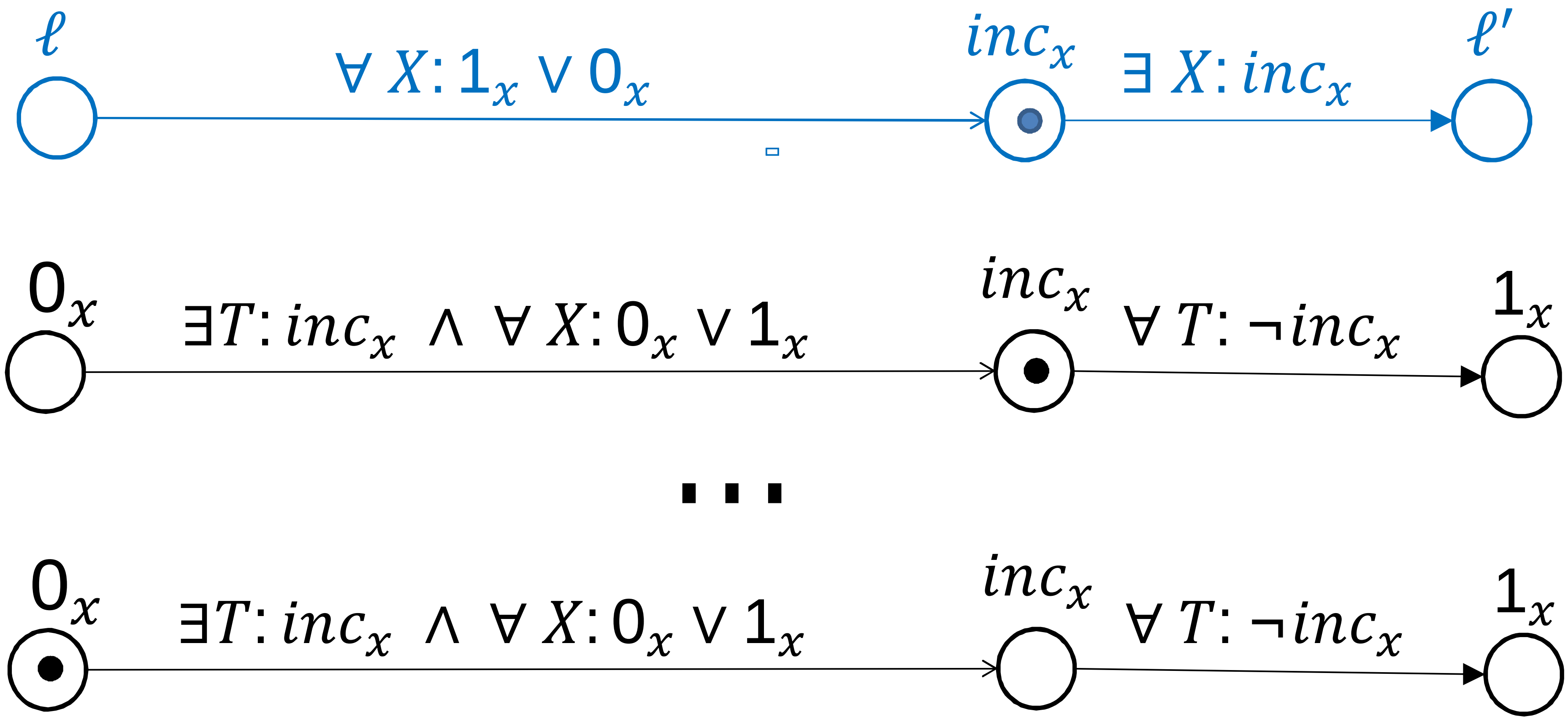

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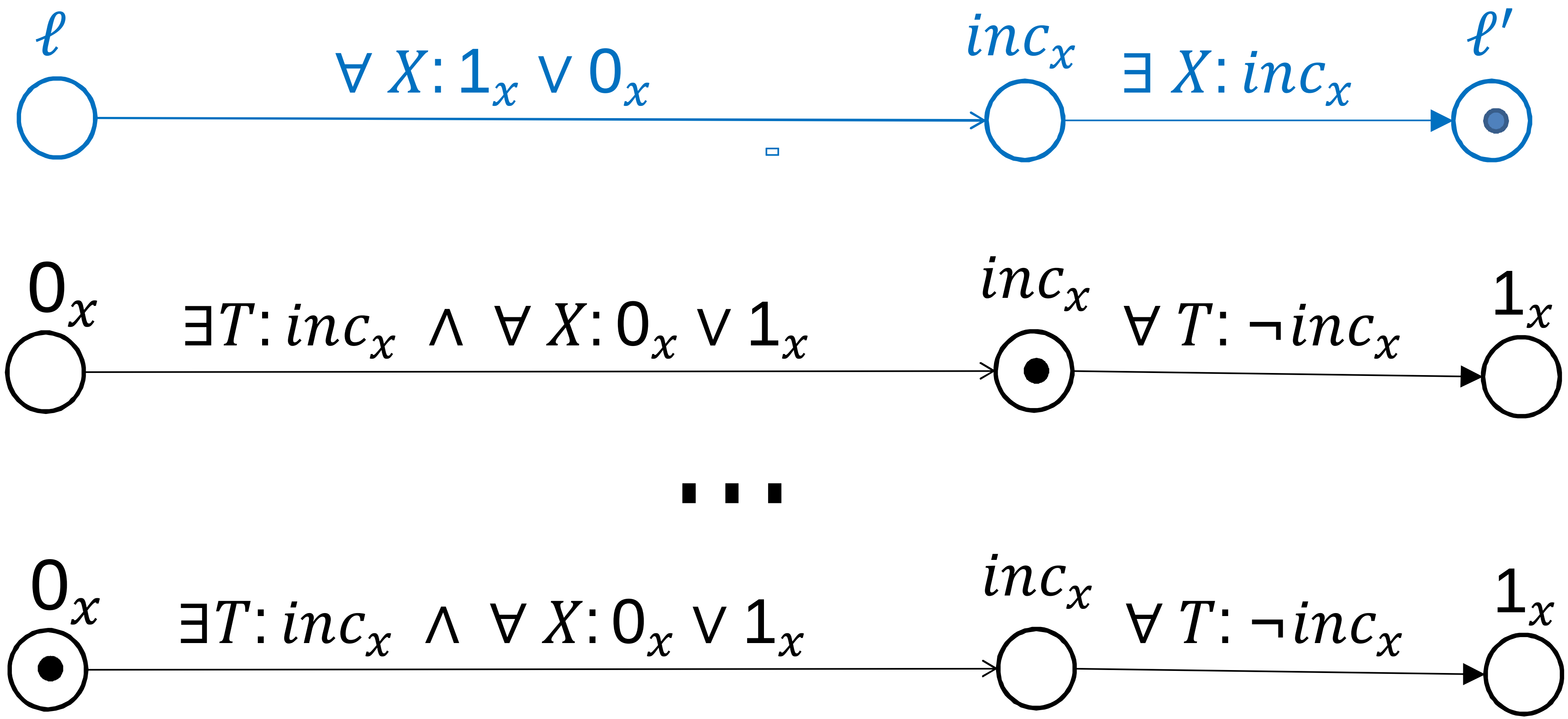

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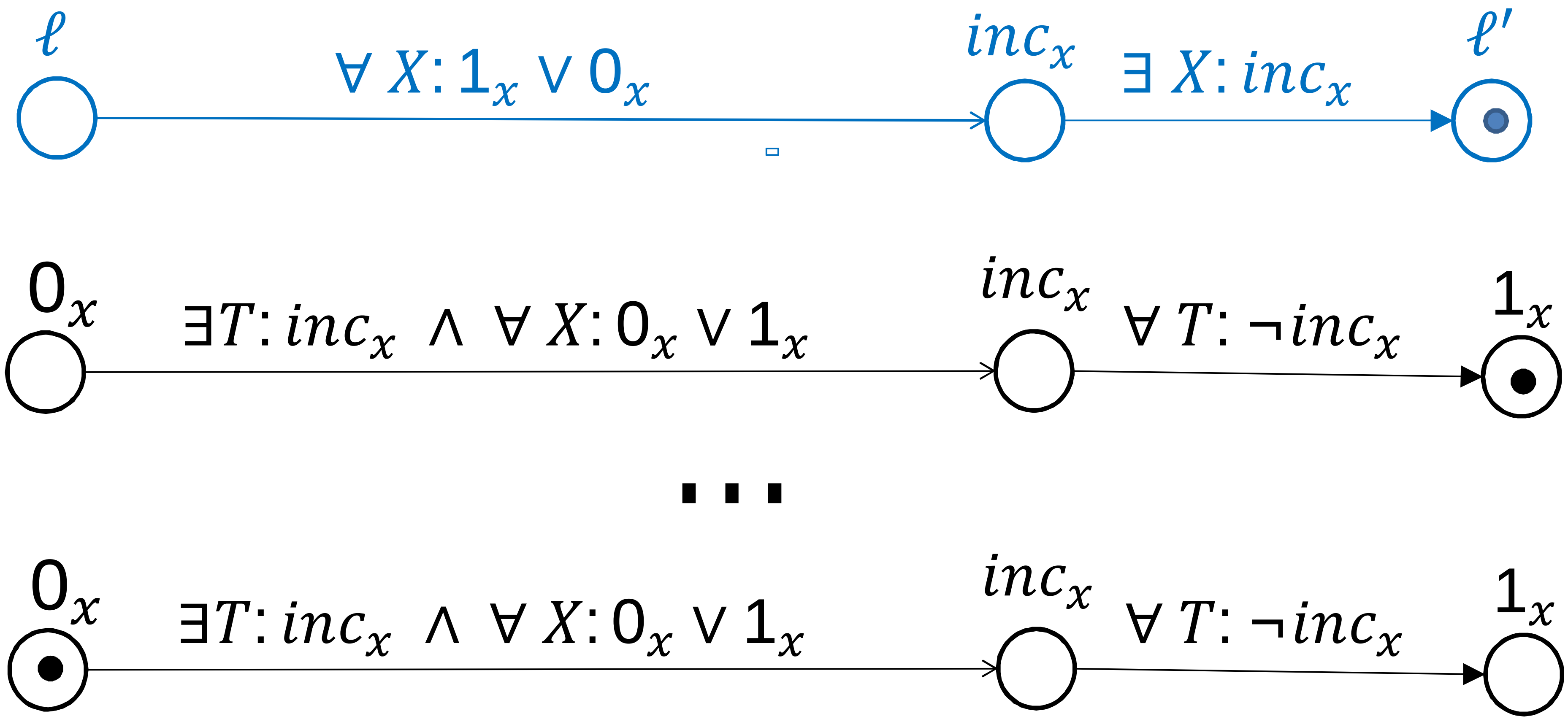
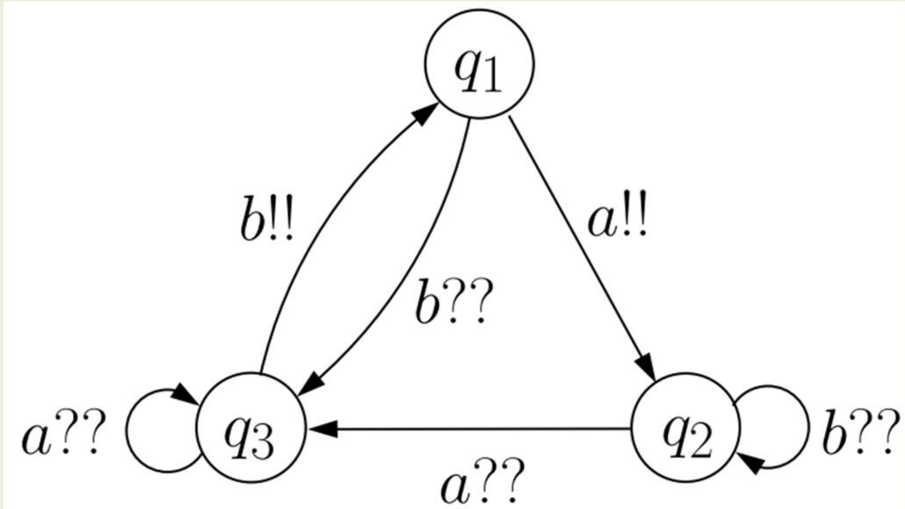


Comm. Mechanisms: Broadcast

Reliable broadcast

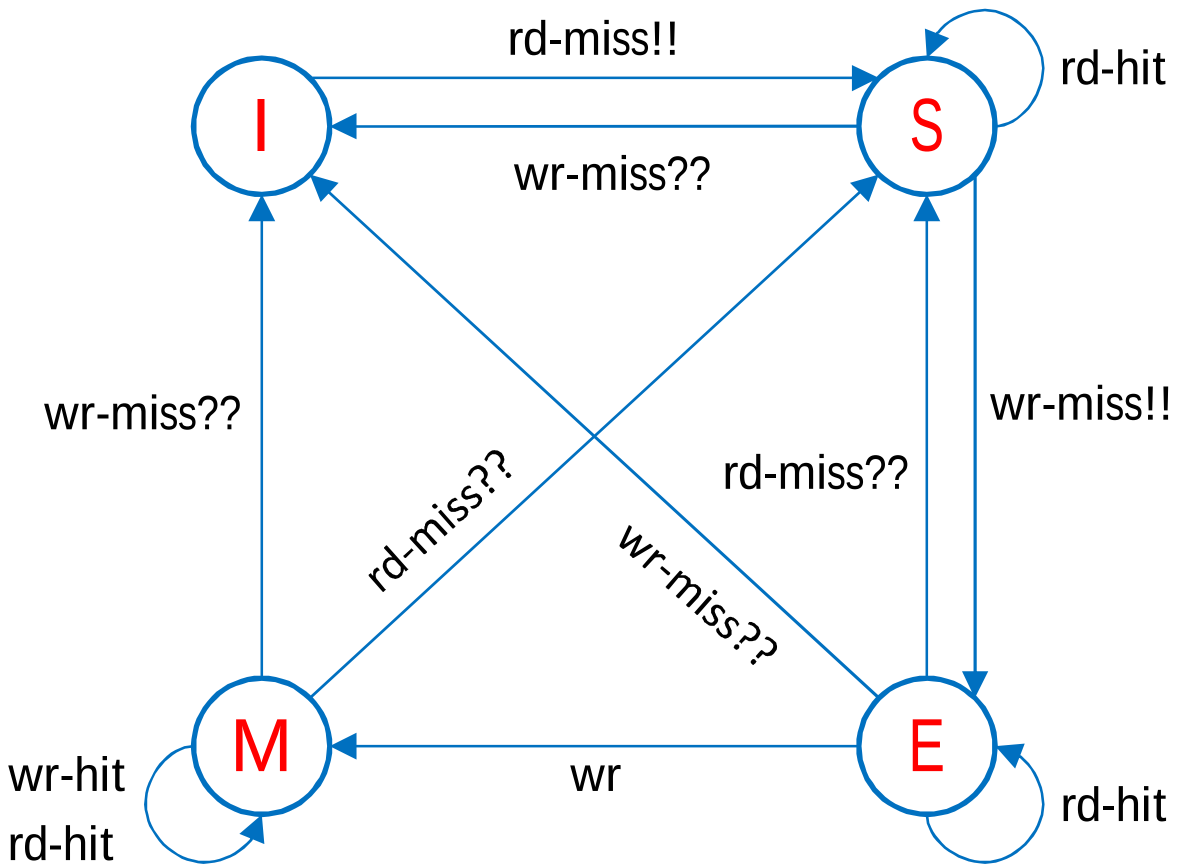
- A process sends a message
- All other processes receive the message (instantaneously)



Examples

- Distributed algorithms
- Hardware protocols (cache-coherence) (Emerson and Kahlon 2003)
- Predicate abstractions of multithreaded programs (Kaiser, Kroening, Liu, Wahl, 2014,2015)

Reliable broadcast



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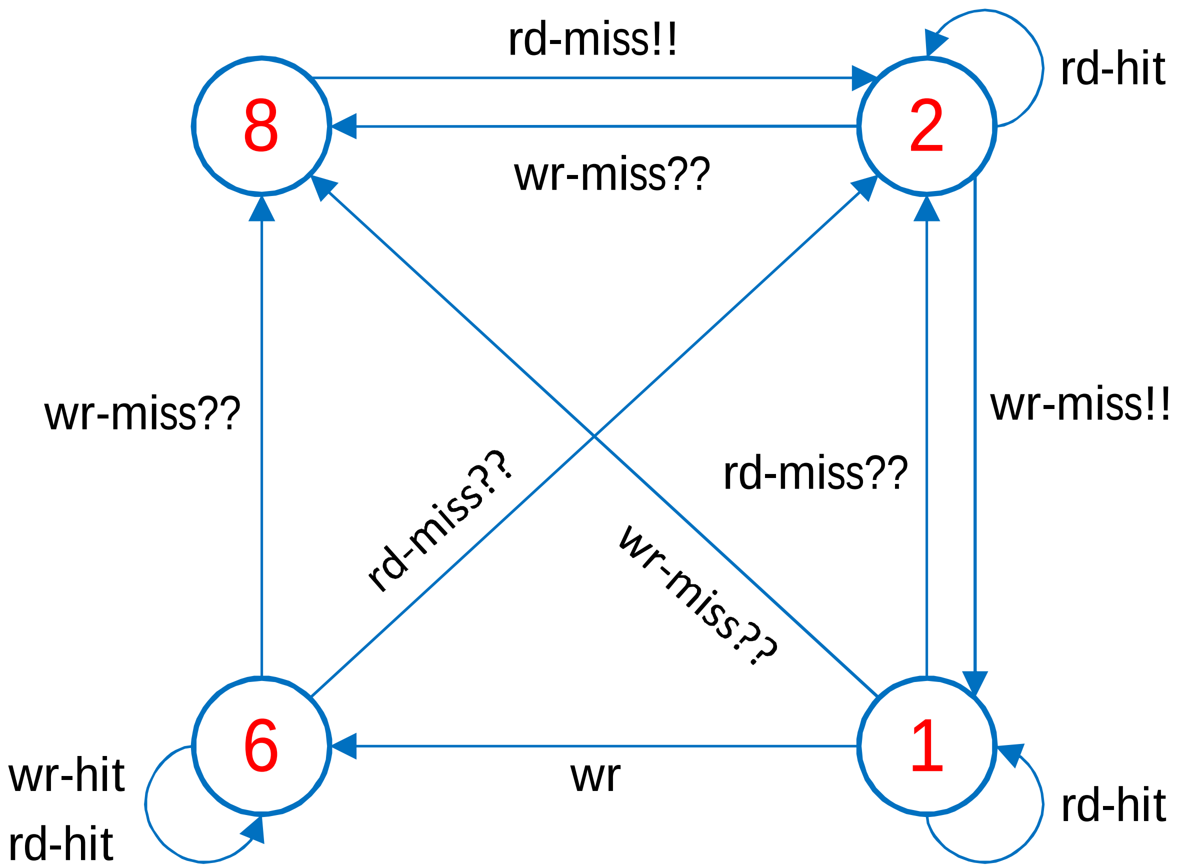

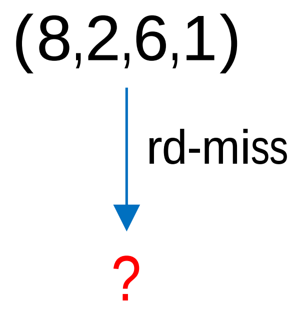

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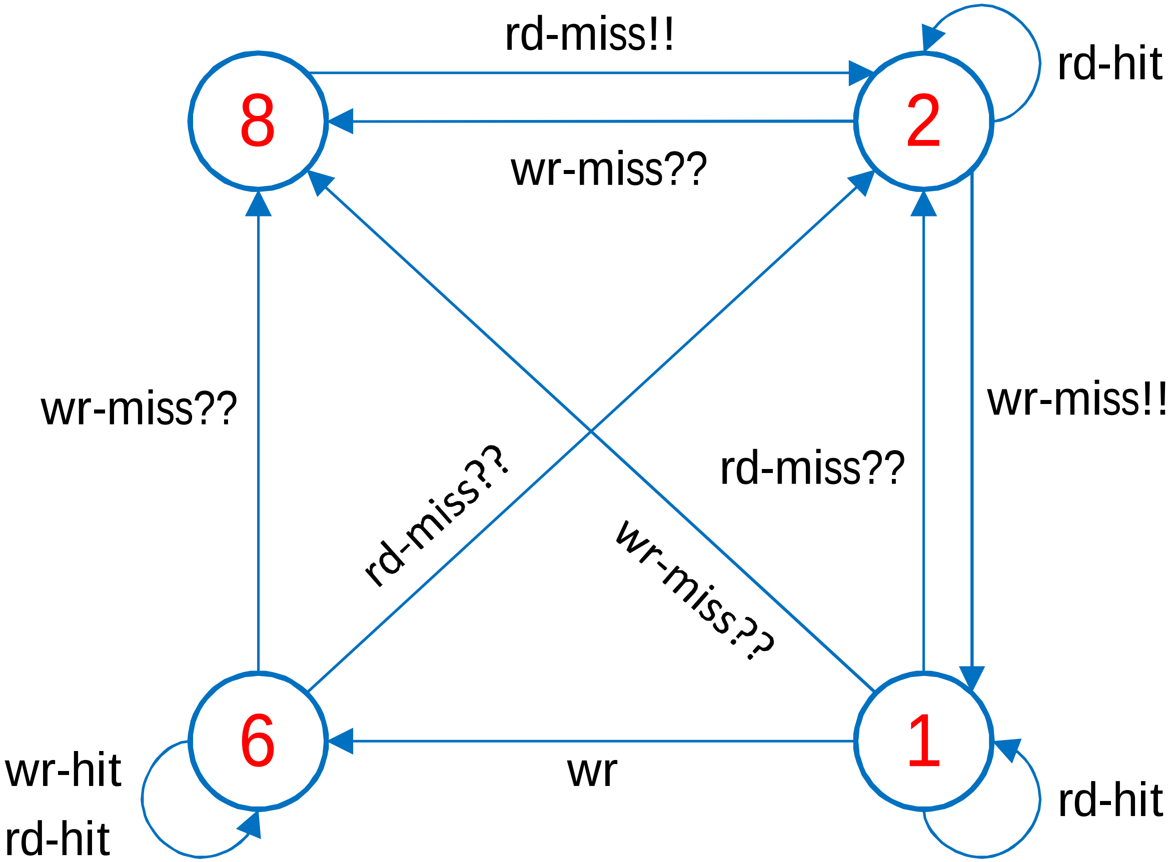
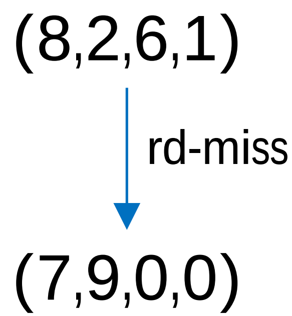




Reliable broadcast

- Theorem [E., Finkel, Mayr 99] The coverability problem for broadcast protocols is decidable.

- Informally:

  Anonymous crowds with local guards

  are not Turing powerful

- Algorithm: Backward Search

  Abdulla *et al*. (LICS'96), based on the theory of well-quasi-orders.

Backward Search

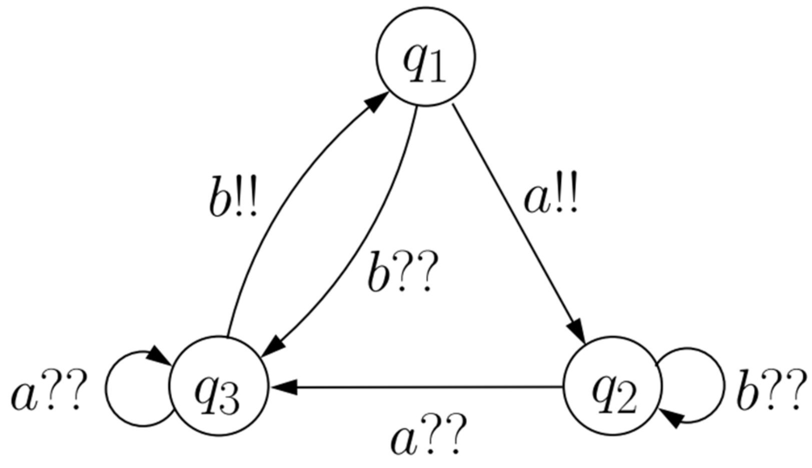
C ≔ set of dangerous conf.

Iterate $C \coloneqq C \cup pre(C)$ **until**

$C \cap Init \neq \emptyset$; **return** „unsafe“

or

C fixpoint; **return** „safe“



Init

$C_1 := C$

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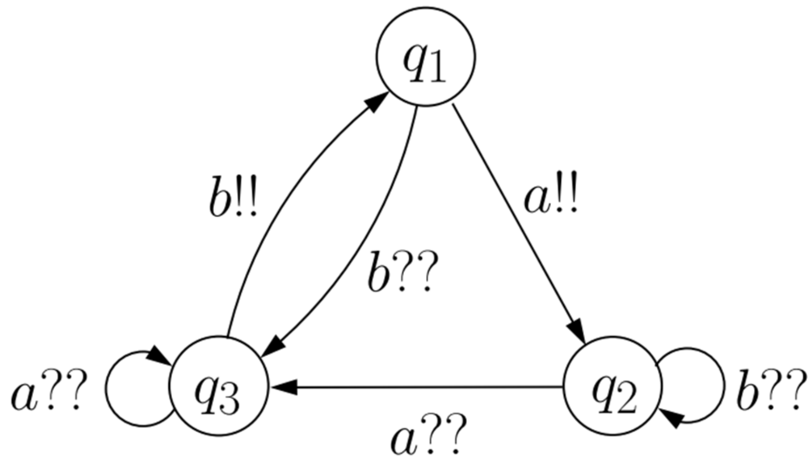

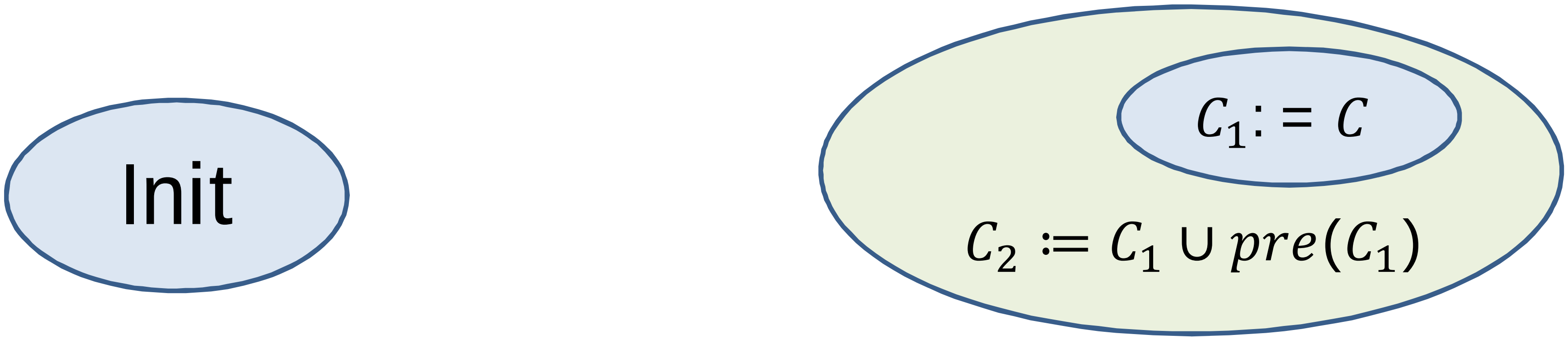

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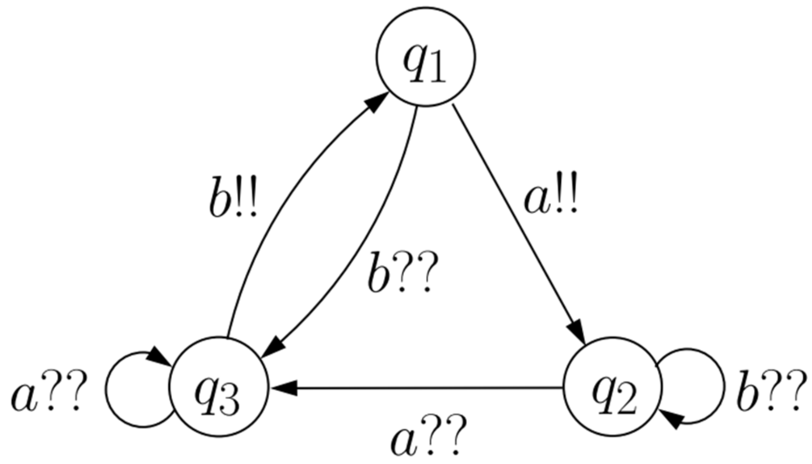

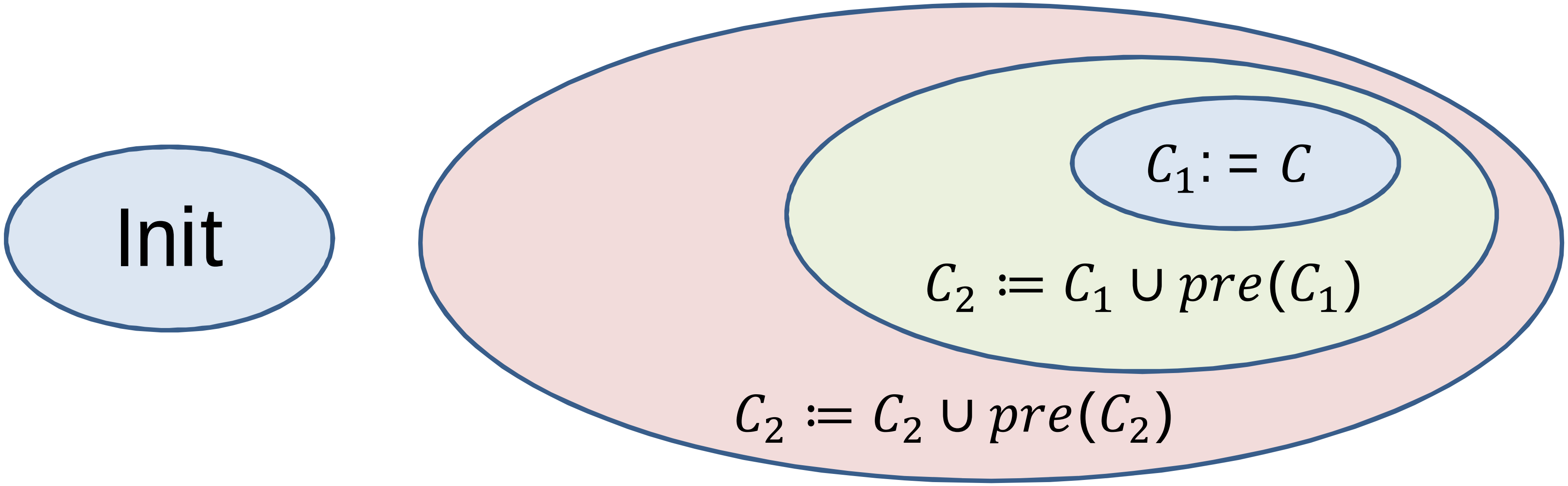

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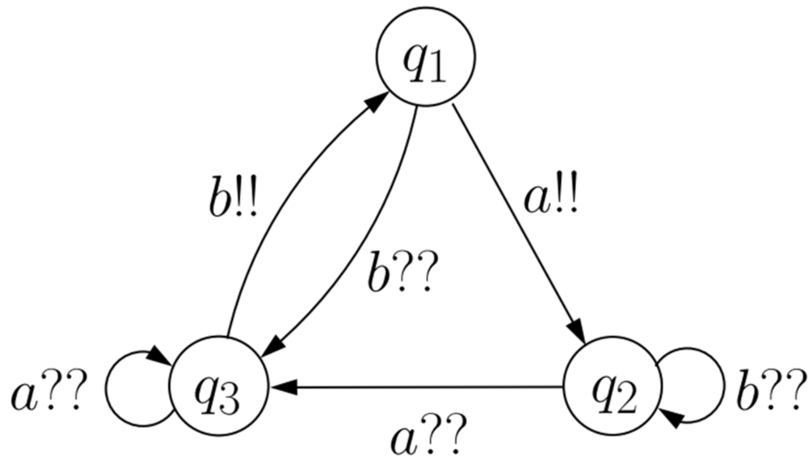

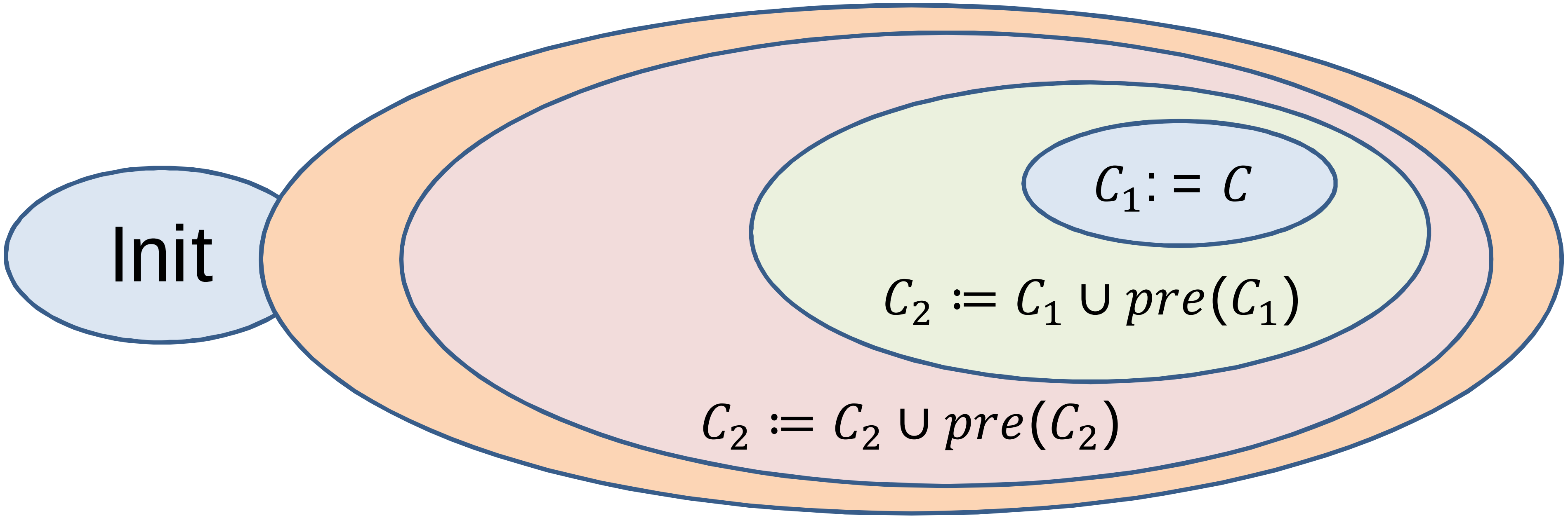

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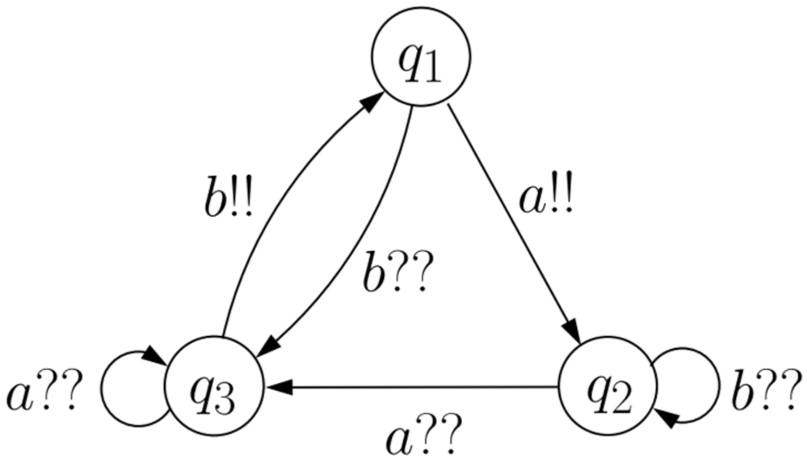

Problems:

- C can hold infinite sets. Finite representation?
- Termination?

Finite representation: Upward-closed sets

- Definition: $c \leq c'$ if c' has at least as many processes as c in each state

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- Observation 3: The union of upward-closed sets is upward-closed.
- Consequence: all sets computed by Backward Search are upward-closed.

Finite representation: Well quasi-orders

- Proposition: $\leq$ is a well-quasi-order: every infinite sequence $c_1, c_2, \ c_3 \cdots$ of configurations contains an infinite chain $c_{i_1} \leq c_{i_2} \leq c_{i_3} \cdots$

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- Consequence: Every upward-closed set has a finite set of minimal elements, and can be represented by it.

Termination of Backward Search

- Observation: Backward Search computes a sequence $C_0 \subseteq C_1 \subseteq C_2 \cdots$ of upward-closed sets of configurations.
- Theorem: $\bigcup_{i \geq 0} C_i = C_j$ for some $j \geq 0$.

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Consequence: There is a family $\{P_n\}_{n=0}^{\infty}$ of broadcast protocols with $O(n)$ states such that the smallest number of processes required to reach the dangerous state is a non-primitive-recursive function of n.

Complexity and cut-off bound

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C

p of

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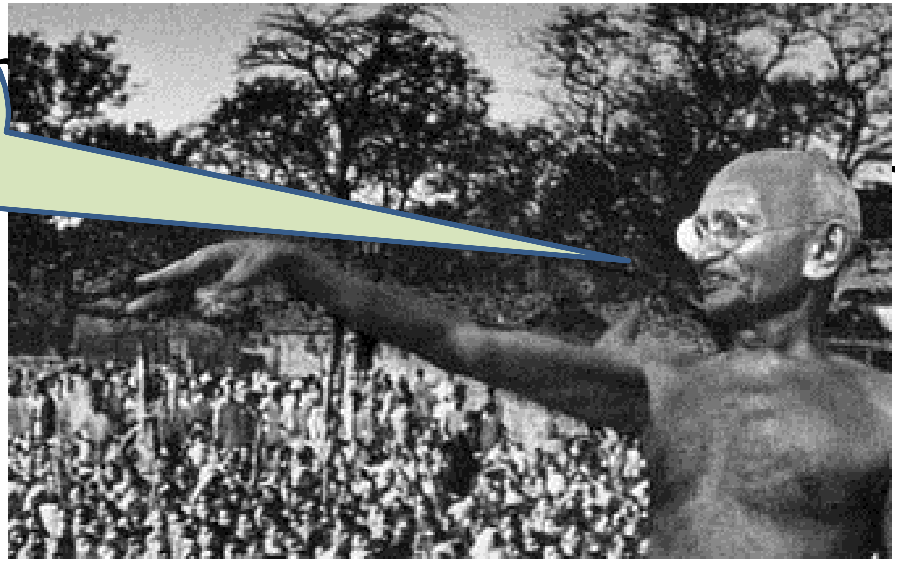

Complexity and cut-off bound

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And yet, backwards reachability is useful for verification! I‘ve used it to prove properties of a dozen cache-coherence protocols: their templates have under 10 states!

G. Delzanno

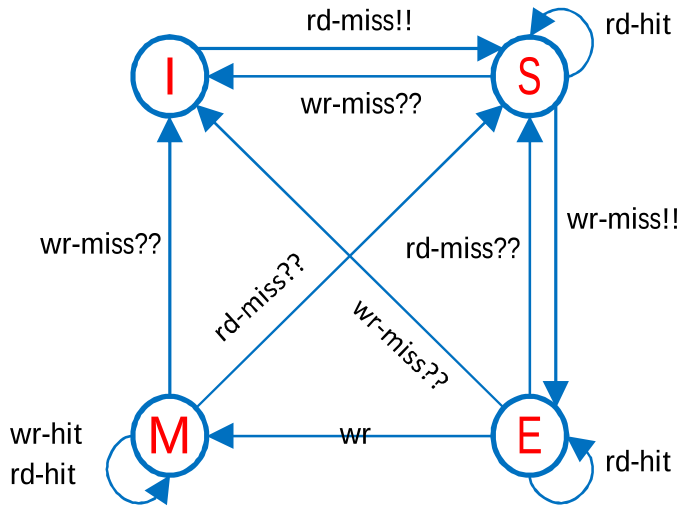
Application to the MESI-protocol

- Are the states M and S mutually exclusive?
- Check if the upward-closed set with minimal element

  m =1, e=0, s=1, i =0

  can be reached from the initial set

  m=0, e=0, s=0, i =N



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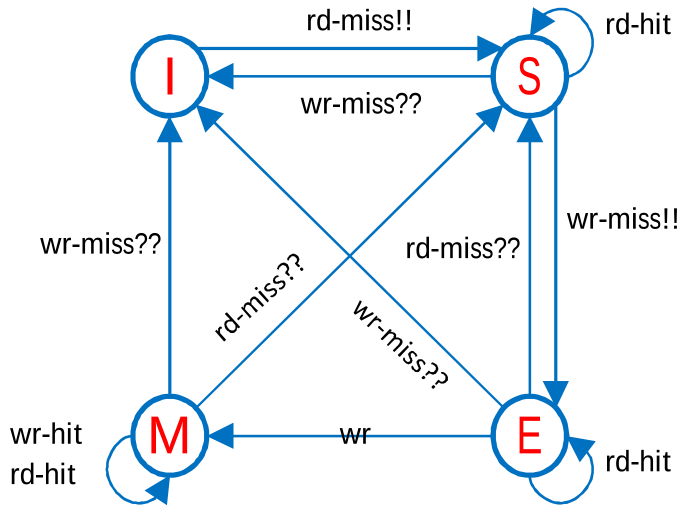


C_0 : $\quad m \geq 1 \wedge s \geq 1$

$C_1 = C_0 \cup pre(C_0)$: $\quad (m \geq 1 \wedge s \geq 1) \vee (m = 0 \wedge e = 1 \wedge s \geq 1)$

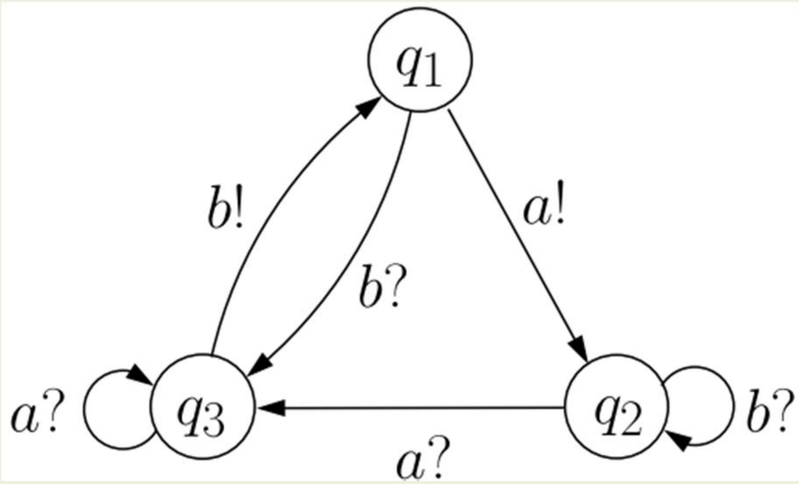
$C_2 = C_1 \cup pre(C_1)$: $\quad C_1$

Comm. Mechanisms: Rendez-Vous

Rendez-vous

- Synchronous exchange of a message between two processes

  (binary encounters)



Examples

- Biochemical systems
- Vehicular networks
- Communication protocols: equivalent to message passing with bounded channels

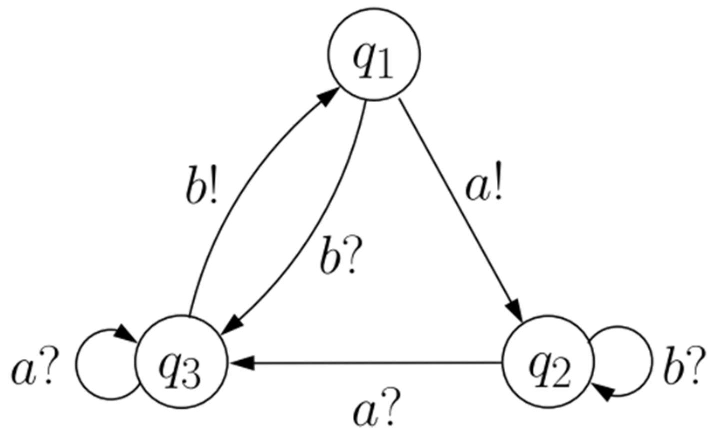
Rendez-vous

- Important differences with broadcast:
  - o no way to reliably reach all processes
  - o the crowd can no longer produce a leader

Theorem:

The coverability problem for rendez-vous communication and is EXPSPACE-complete.

The problem for leaderless perametrized configuration can be solved in PTIME.



Lower bound

[Lipton 1976]

A template with $O(n)$ states can simulate a counter counting up to 2^{2^n}.

Simulating counter machines

- “ x has value n” modelled by n processes in state 1_c

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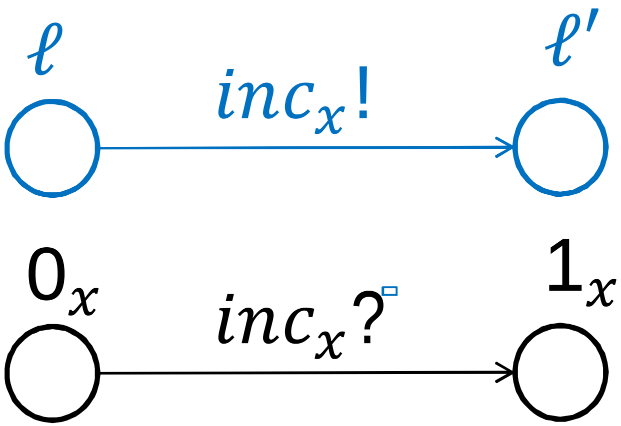
- $\ell: x \coloneqq x + 1; \ell'$ ... easy to simulate: rendez-vous between a leader and a "unit process"



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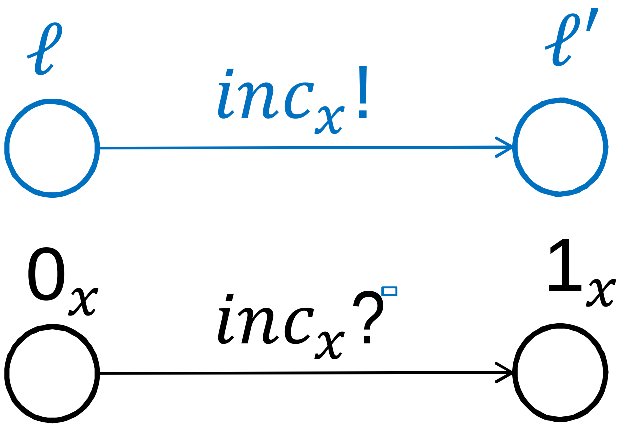
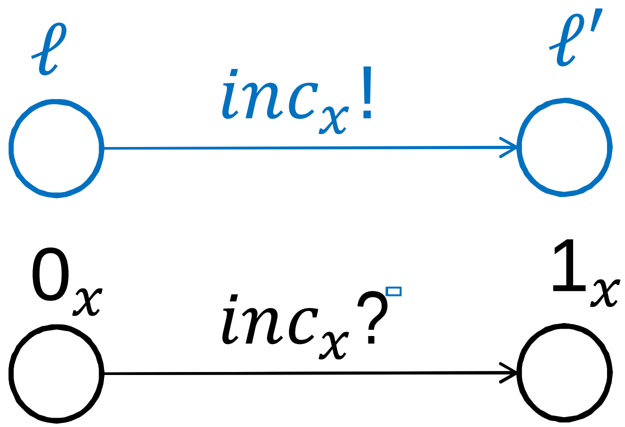


- $\ell: x \coloneqq x - 1; \ell'$ ... similar

Simulating counter machines

- "x has value n" modelled by n processes in state 1_c
- $\ell: x := x + 1; \ell'$ ... easy to simulate: rendez-vous between a leader and a "unit process"



- $\ell: x := x - 1; \ell'$ ... similar
- Problem: simulate ℓ: if $x = 0$ then goto ℓ' else goto ℓ''
  Rendez-vous cannot check that no process is in state 1_x

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- Idea: Simulate only counters counting up to k
- Introduce for each counter x a complementary counter $\overline{x}$, and maintain the invariant $x + \overline{x} = k$

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$$\ell: \quad \text{goto } \ell_1 \text{ or goto } \ell_2$$

$$\ell_1: \ \overline{x} := \overline{x} - k;\ \overline{x} := \overline{x} + k;\ \text{goto } \ell'$$

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- The execution of this code can get stuck, but if it terminates it faithfully simulates the zero-test.

Simulating counter machines

- Idea: Simul … k
- Introduce … tary counter $\overline{x}$, and mai…
- Simulate … ℓ'' by a nondeterm…

Cheat!!! You can't directly simulate me! You only know how to simulate $\overline{x} \coloneqq \overline{x} - 1$!

ℓ: go… $_1$ or goto ℓ_2

ℓ_1: $\overline{x} \coloneqq \overline{x} - k$; $\overline{x} \coloneqq \overline{x} + k$; goto ℓ'

ℓ_2: $x \coloneqq x - 1$; $x \coloneqq x + 1$; goto ℓ''

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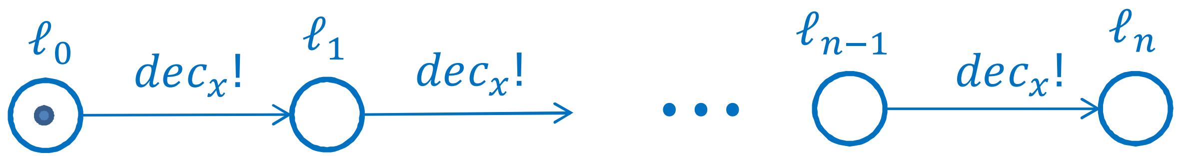
- Question: By how large a number k can templates with a total of $\mathrm{O}(n)$ states decrease a counter $\overline{x}$?

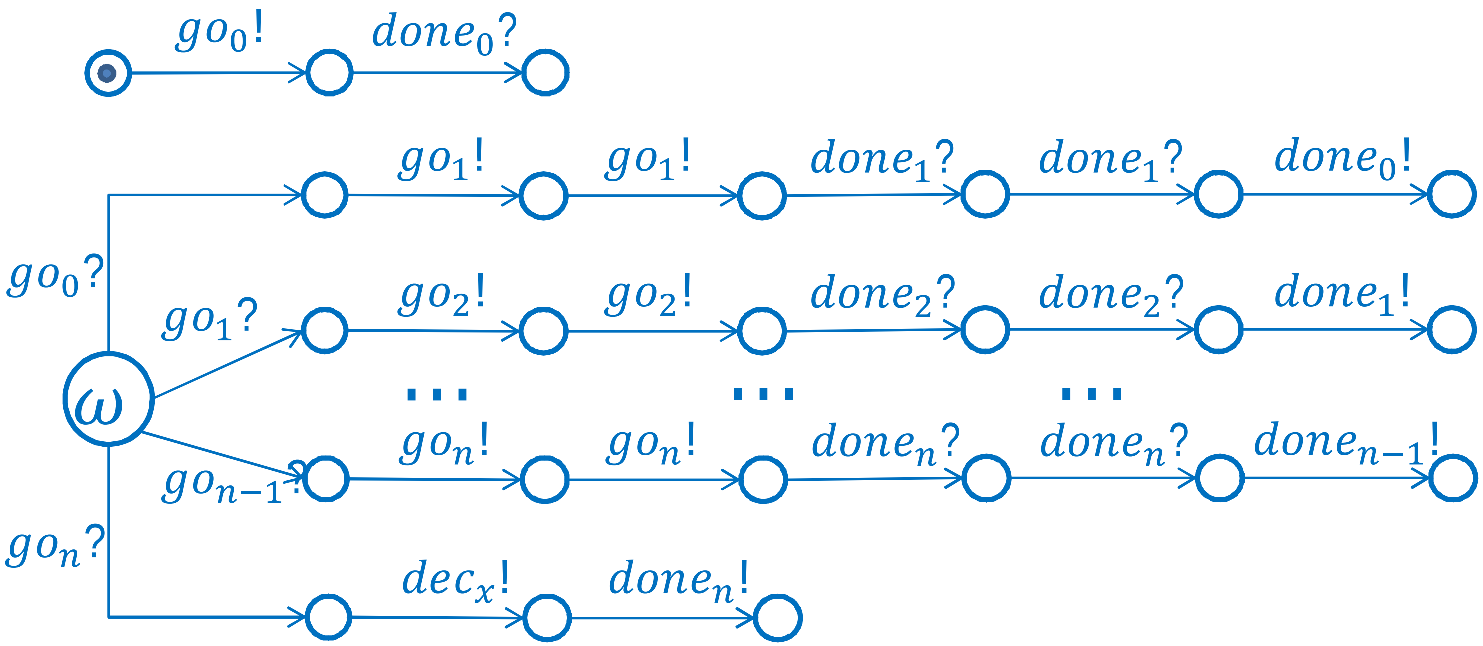
Simulating counter machines

- Question: By how large a number k can templates with a total of $O(n)$ states decrease a counter $\overline{x}$?
- First answer: $k = O(n)$
  - One leader template:



Simulating counter machines

- Question: By how large a number k can templates with a total of $O(n)$ states decrease a counter $\bar{c}$?
- Better answer: $k = 2^{O(n)}$
    - Iterated doubling:



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  - Iterated squaring

Iterative squaring

- Given templates simulating $\overline{\boldsymbol{x}} \coloneqq \overline{\boldsymbol{x}} - \boldsymbol{m}$ as

  for $i = m$ to 1

  $\overline{x} \coloneqq \overline{x} - 1$

  endfor

  Lipton constructs templates simulating $\overline{\boldsymbol{x}} \coloneqq \overline{\boldsymbol{x}} - \boldsymbol{m}^2$ as

  for $j = m$ to 1

  for $i = m$ to 1

  $\overline{x} \coloneqq \overline{x} - 1$

  endfor

  endfor

Upper bound

Lower bound [Lipton 1976]

Upper bound [Rackoff 1978]:

If the goal state is coverable, then it is coverable in an instance with 2^{2^n} processes.

Upper bound

- Fix a template T with n states $q_1, \ldots, q_n$
- For every configuration c of T (initial or not!), let $\ell(c)$ be the length of the shortest sequence covering q_1 (if it exists).
- Let $f(n)$ be the maximum of all $\ell(c)$.
- How fast can $f(n)$ grow with n ?

Upper bound

- Define $g(n, i)$ as $f(n)$, but starting from "ω-configurations" of the form $(k_1, \dots, k_i, \omega, \dots, \omega)$ where ω means „arbitrarily many".

  We are interested in $g(n, n) = f(n)$
- We have $g(n, 0) = 1$
- Rackoff shows by induction:

$$g(n, i) = g(n, i-1)^i + g(n, i-1)$$

- A little math gives $g(n, n) \leq 2^{2^n}$

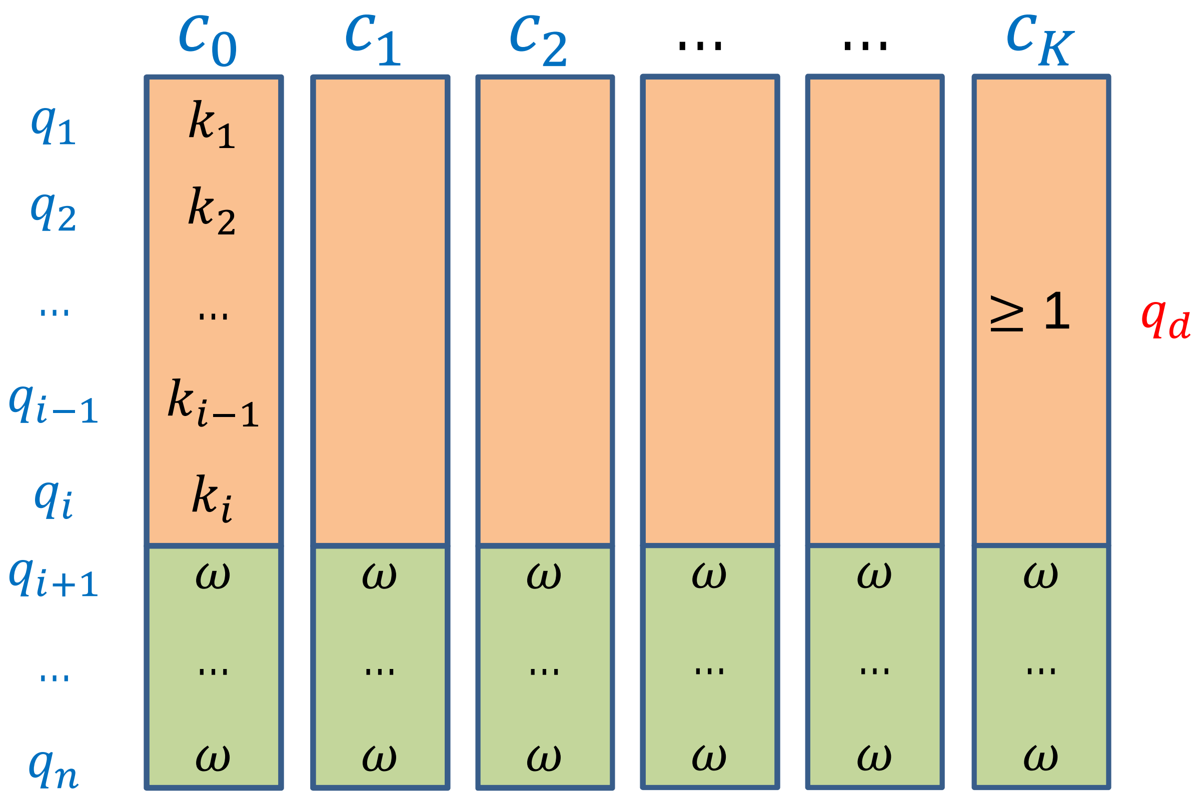
Upper bound

Given a sequence $c_0 \to c_1 \to \cdots \to c_K$ such that

- $c_0 = (k_1, \ldots, k_i, \omega, \ldots, \omega)$ and c_K covers q_d,

find another sequence $c_0 \to c_1' \to \cdots \to c_{K'}'$ such that

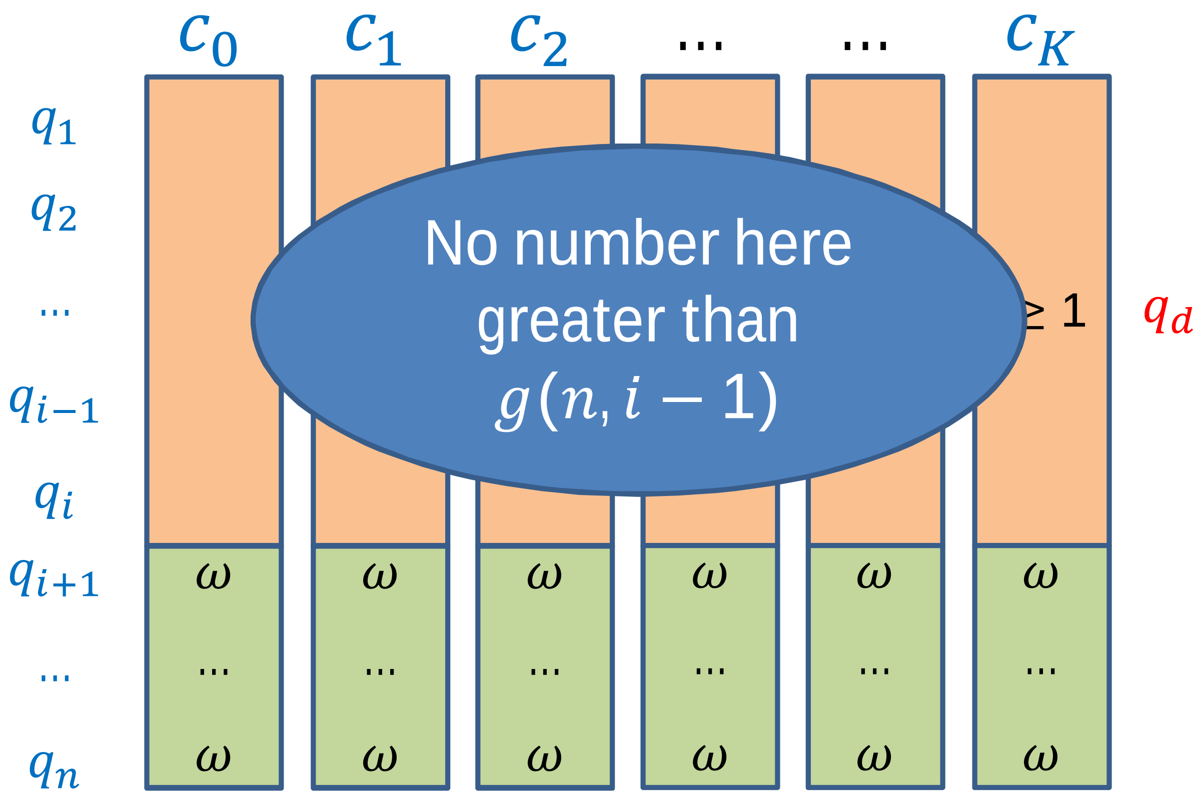
- $c'_{K'}$ covers q_d , and $K' \leq g(n, i) = g(n, i-1)^i + g(n, i-1)$



All c_i different

Upper bound

Case 1



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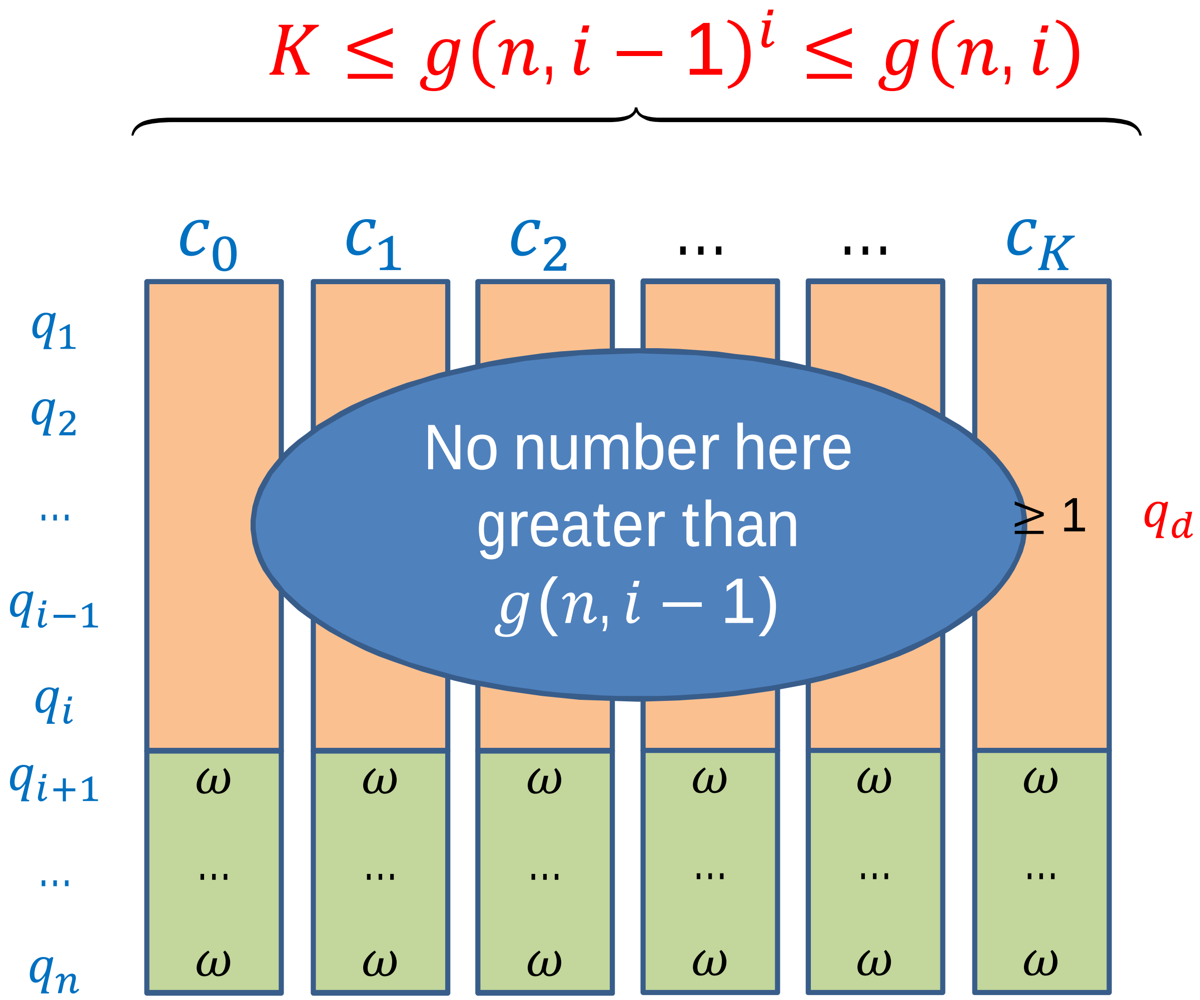
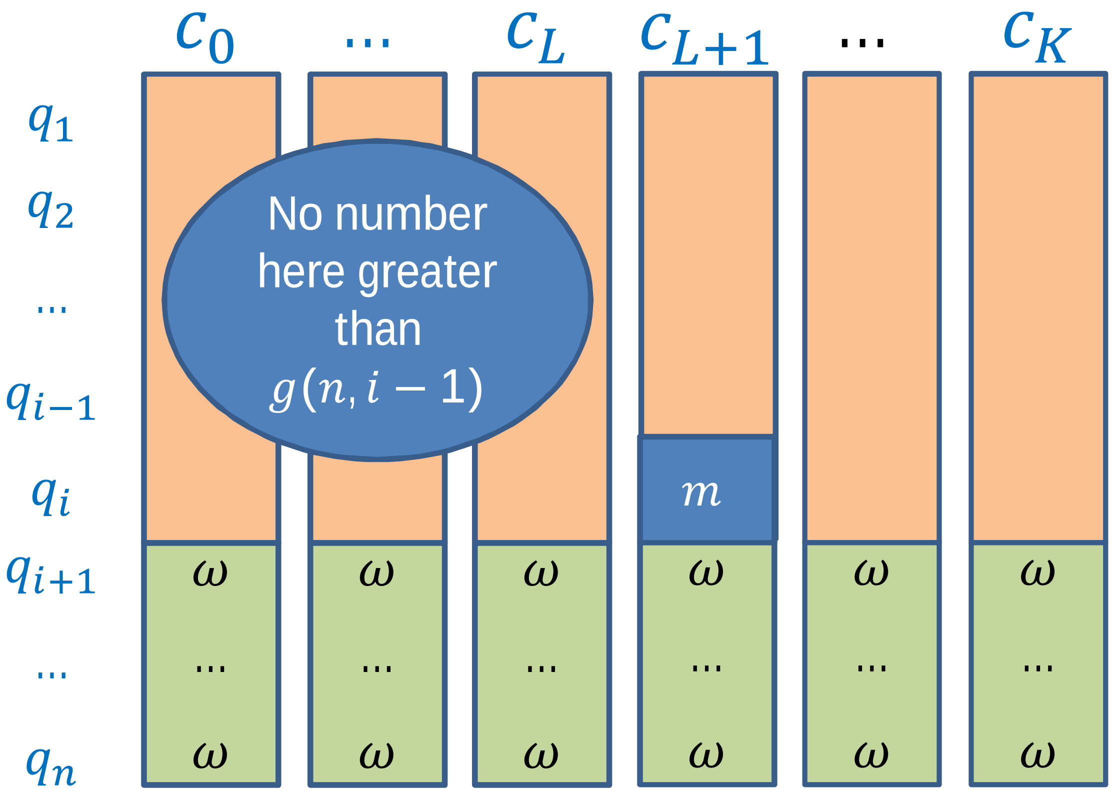


Upper bound

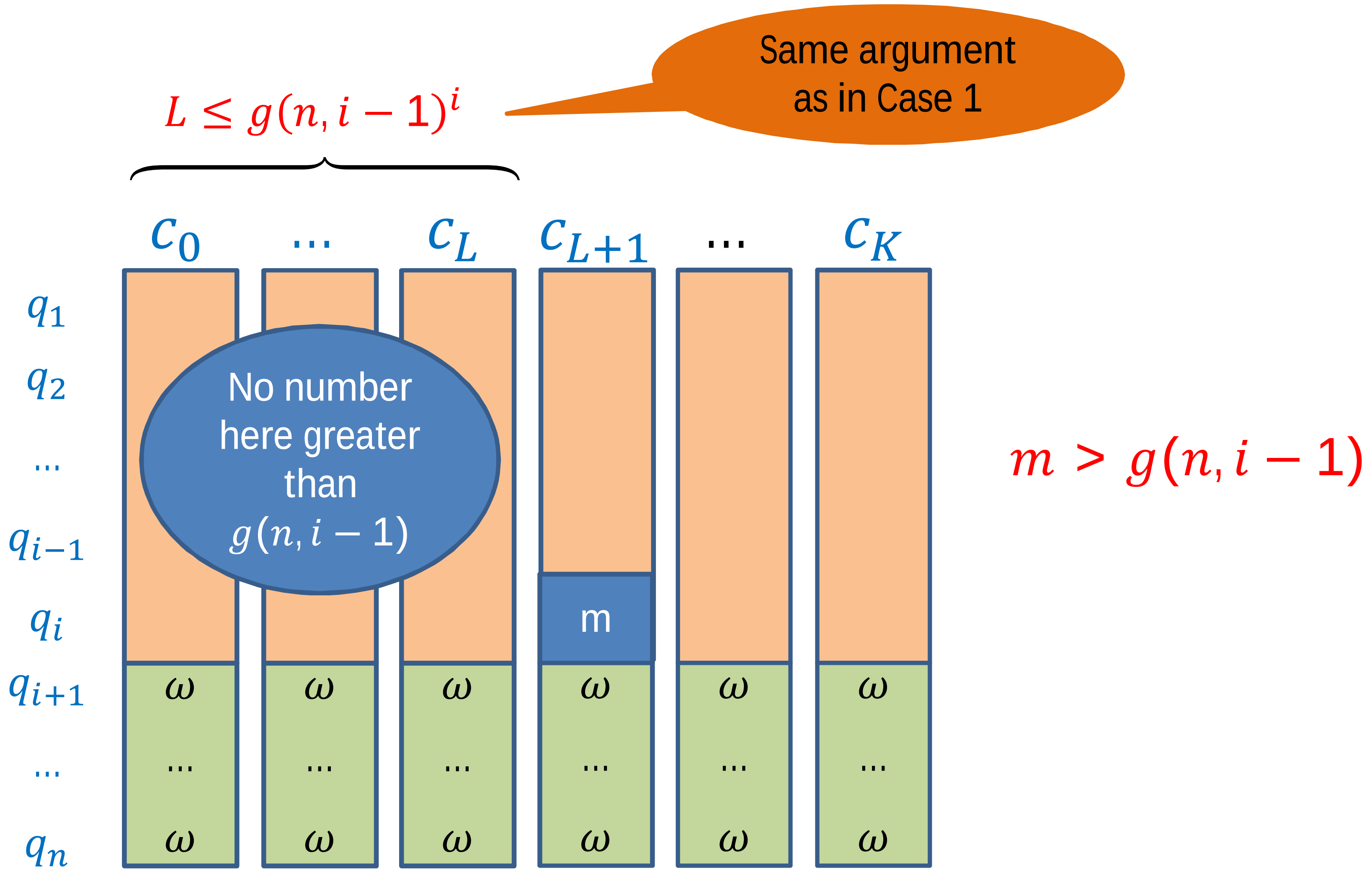
Case 2



$m > g(n, i-1)$

Upper bound

Case 2



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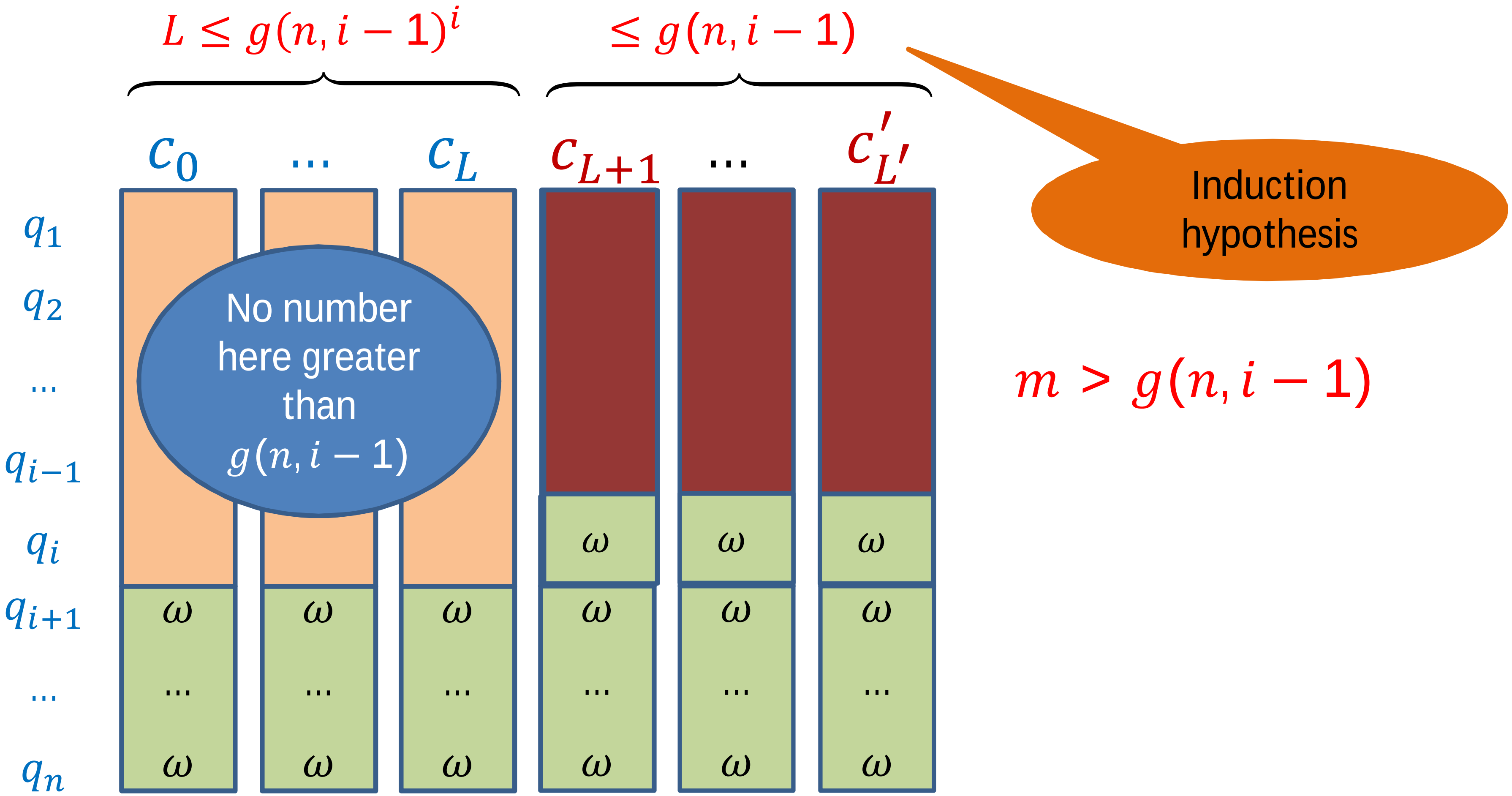

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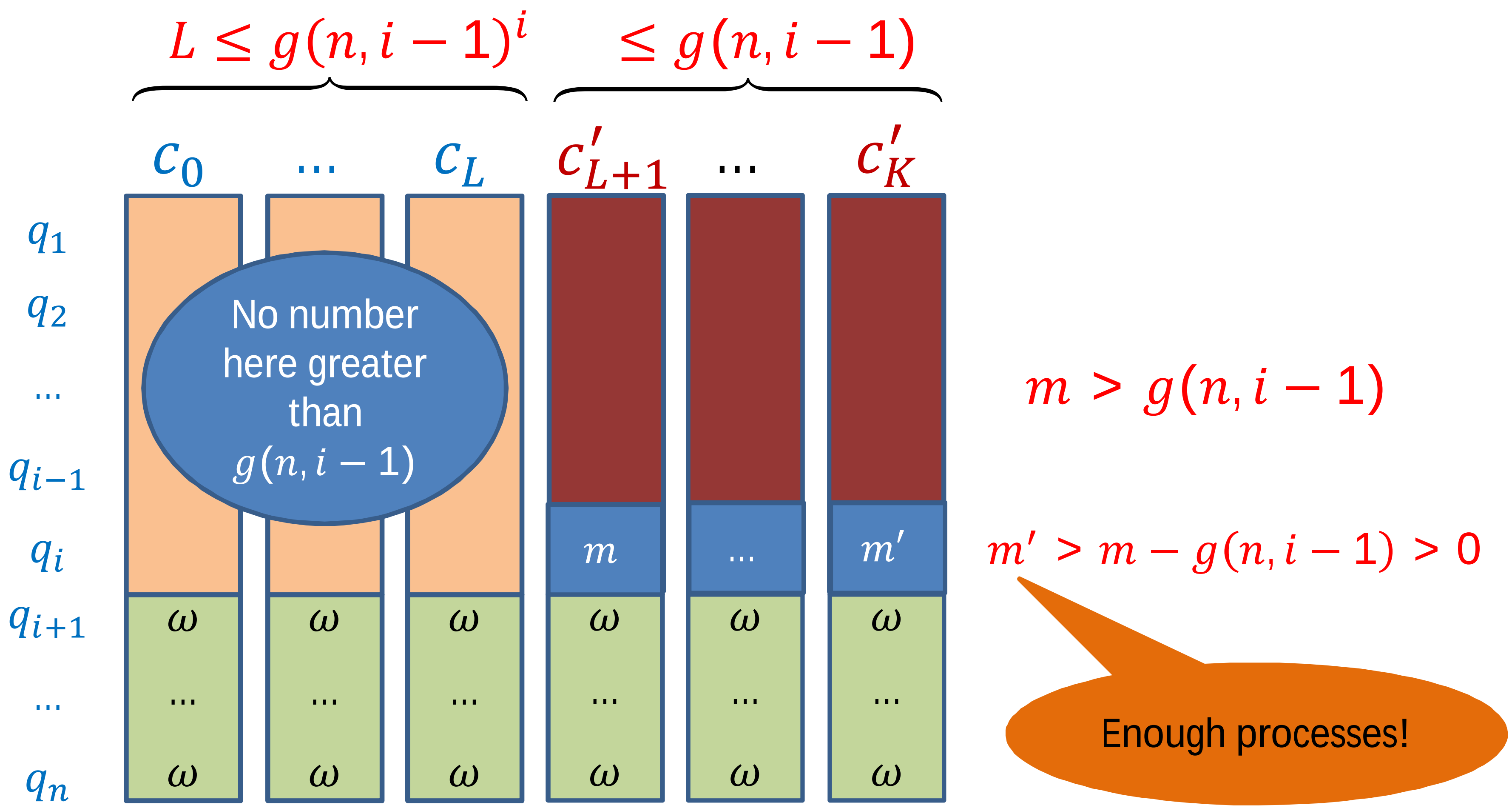

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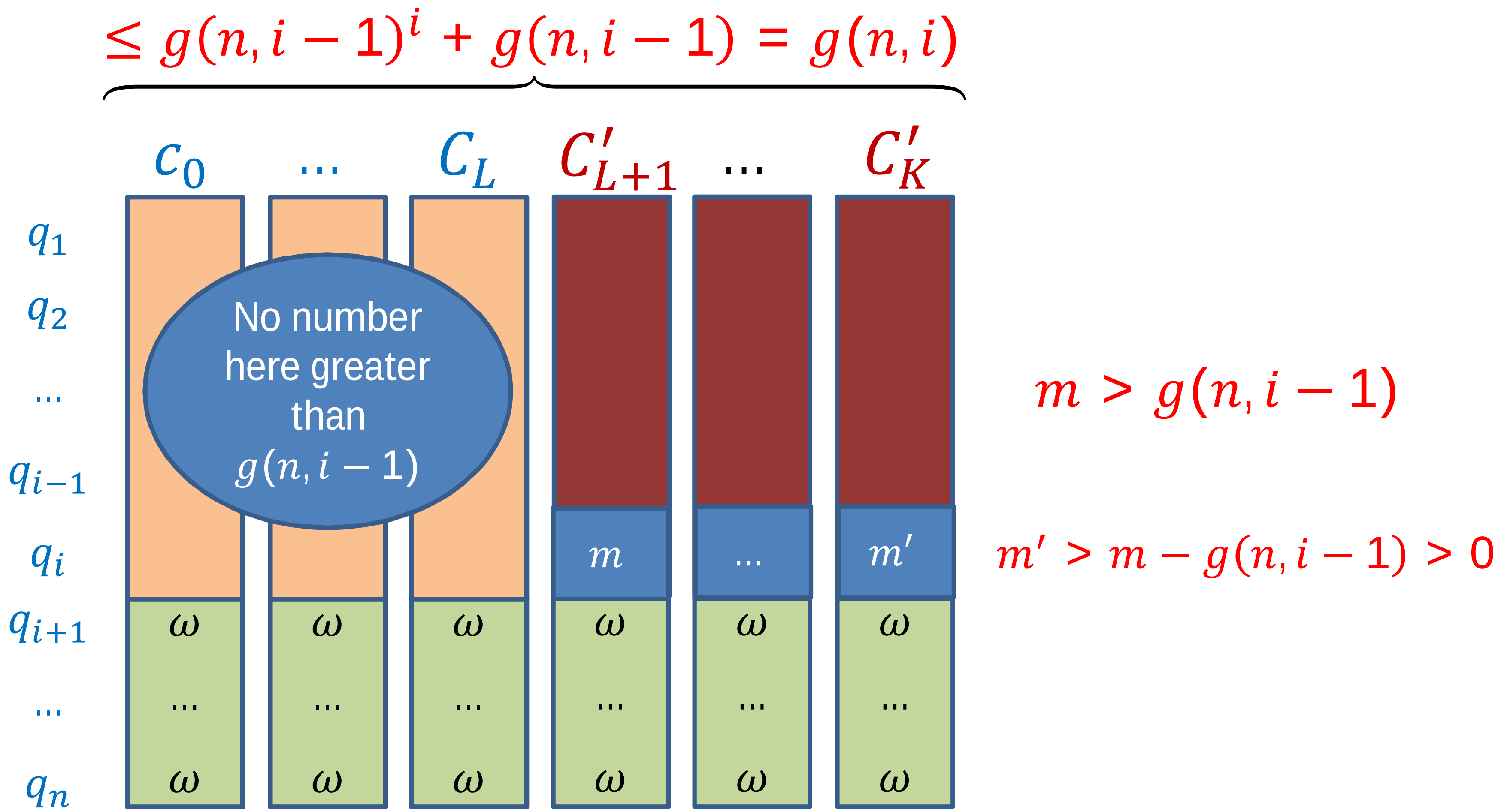
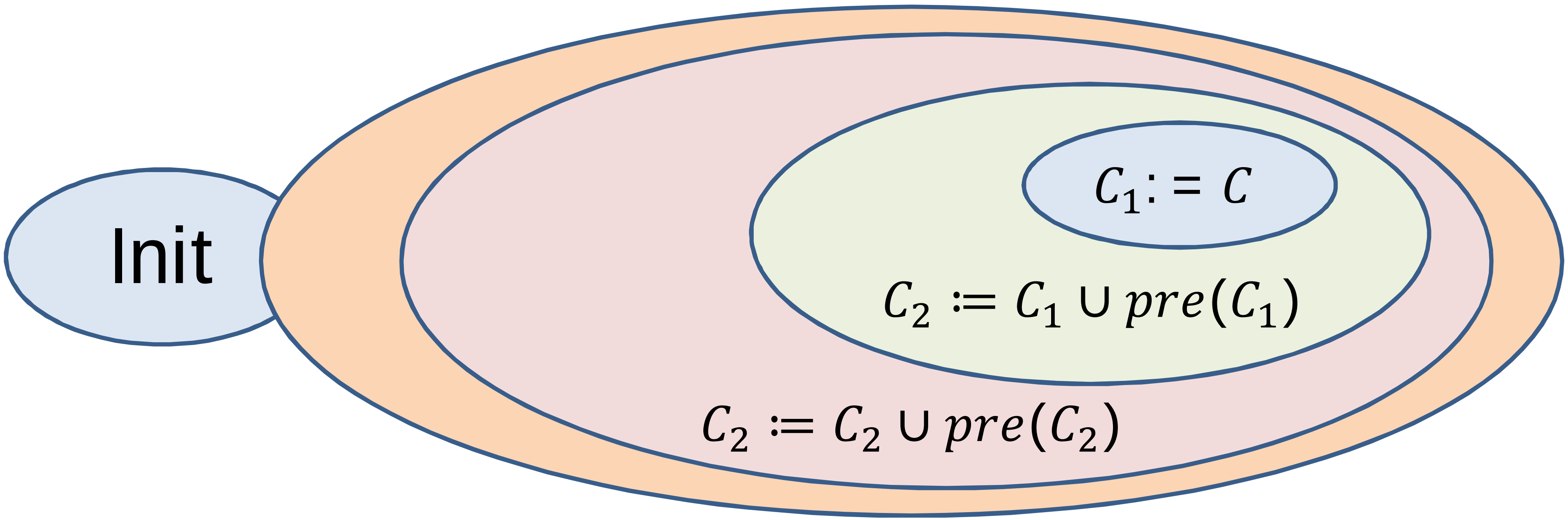


Rendez-vous

Unfortunately, for us verifiers this upper bound is algorithmically pretty useless ...

Backwards Reachability for Rendez-Vous

Theorem [Bozzelli, Ganty 2012]: Backwards reachability runs in double exponential time for rendez-vous systems.



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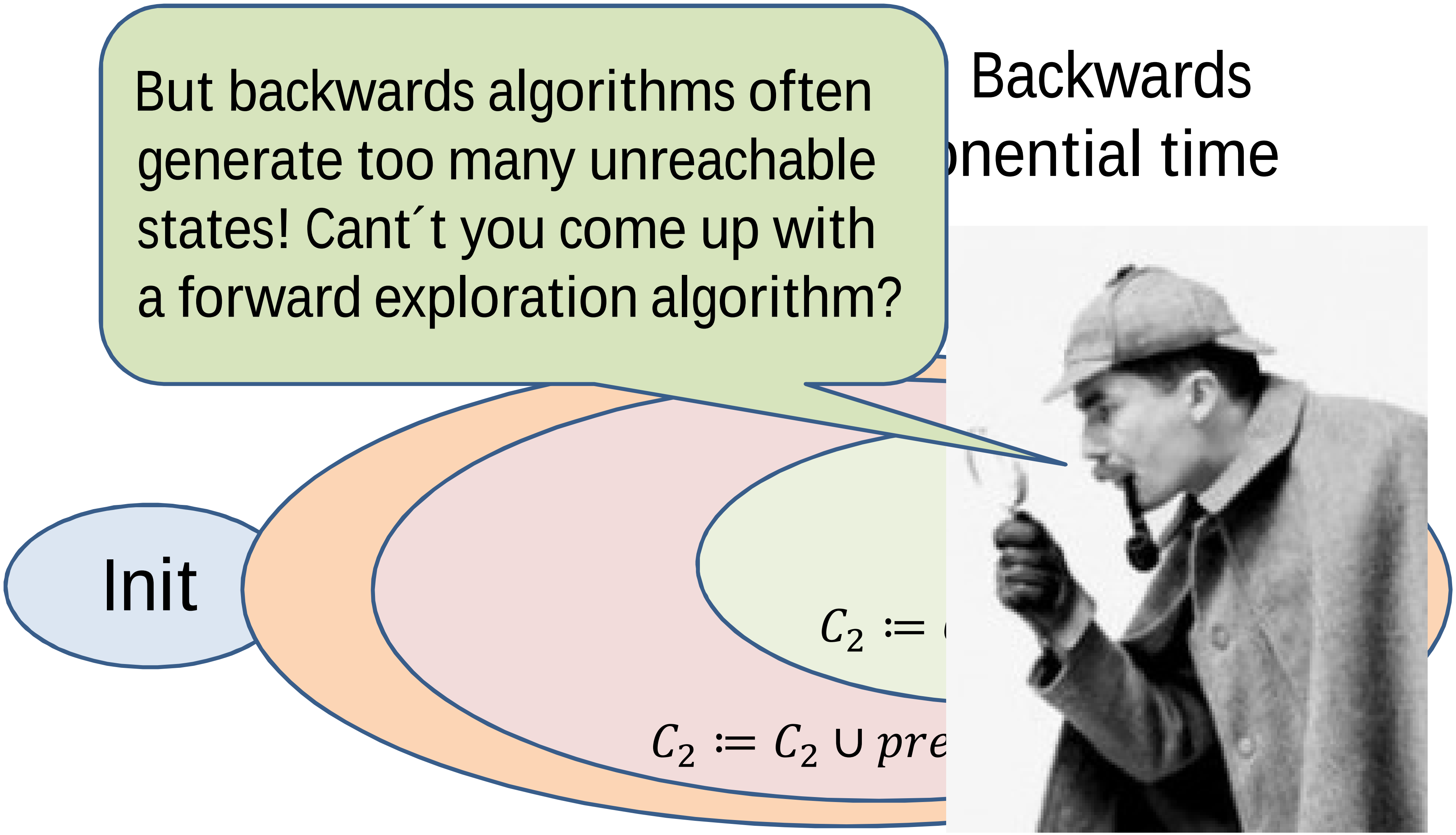
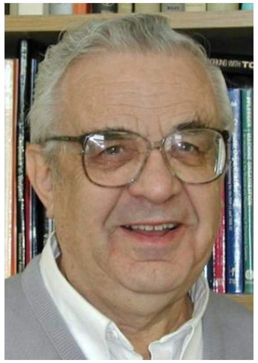


Forward Algorithms

The Karp-Miller coverability graph (1969).

- Graph with ω-configurations as nodes



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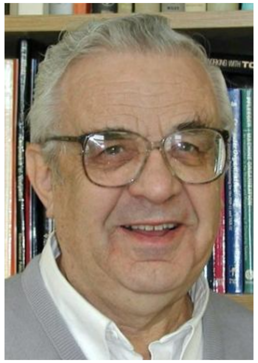
  for $\boldsymbol{\mathcal{C}} = (1,0,N_1,N_2)$: $(1,0,\omega,\omega)$



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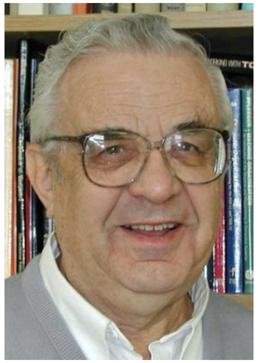

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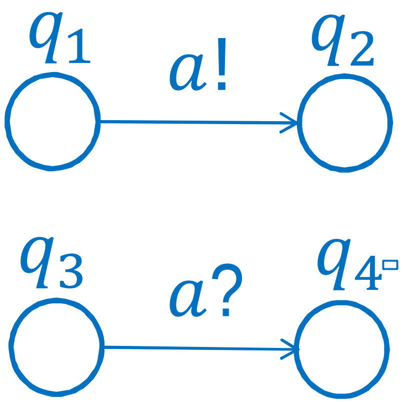

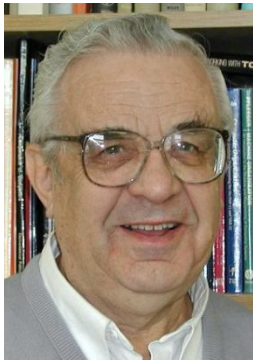

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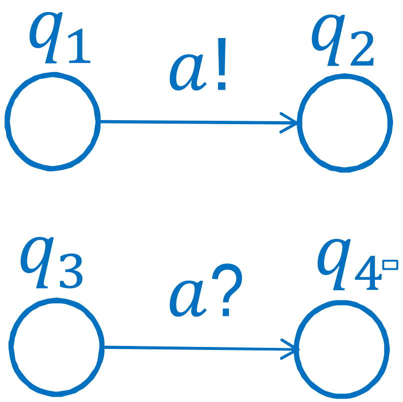
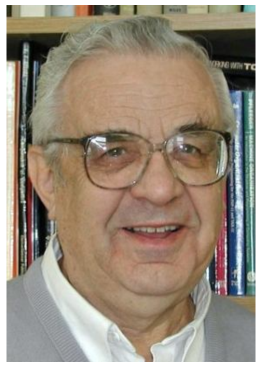


$$(1,0,2,1) \xrightarrow{a} (0,1,1,2)$$



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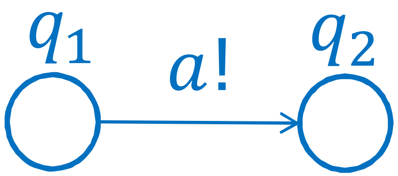
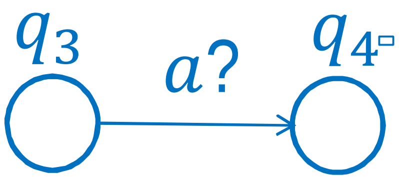
- Initial ω-configuration

  for $\boldsymbol{\mathcal{C}} = (1,0,N_1,N_2)$: $(1,0,\omega,\omega)$
- Construct a „forward reachability graph“ using $\omega - 1 = \omega$



$$(1,0,2,1) \xrightarrow{a} (0,1,1,2)$$



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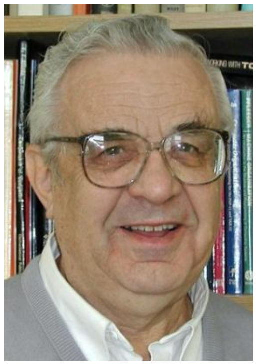


Forward Algorithms

The Karp-Miller coverability graph (1969).

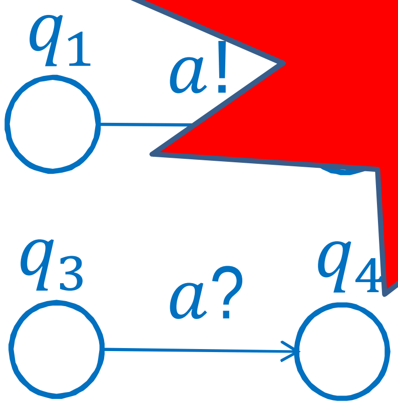
- Graph with ω-configurations as nodes
- Initial ω-configuration

  for $\mathcal{C} = (1,0,$ $N_2)$:

- Construct a

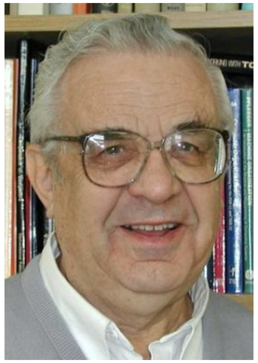
  using ω

Problem:
termination!!



$1) \rightarrow (0,1,1,2)$

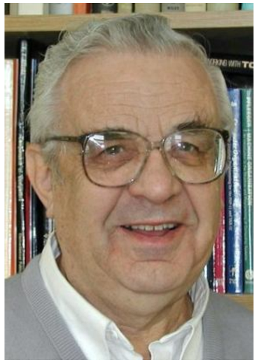
$(1,0,\omega,1) \xrightarrow{a} (0,1,\omega,2)$



Forward Algorithms

- Karp-Miller graph: „Accelerate“ the construction:

$$\begin{aligned}
&\text{If} \quad (\omega,1,1,0) \to \cdots \to (\omega,2,1,2)\\
&\text{then } (\omega,1,1,0) \to \cdots \to (\omega,2,1,2)\\
&\qquad\qquad\qquad\qquad \to \cdots \to (\omega,3,1,4)\\
&\qquad\qquad\qquad\qquad \to \cdots \to (\omega,4,1,6)\\
&\qquad\qquad\qquad\qquad\quad \cdots
\end{aligned}$$



Forward Algorithms

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If $\quad (\omega, 1, 1, 0) \rightarrow \cdots \rightarrow (\omega, 2, 1, 2)$

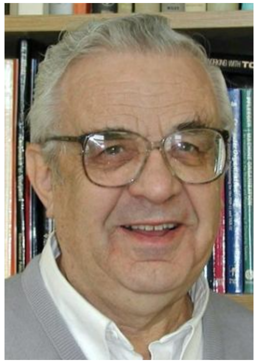
then $(\omega, 1, 1, 0) \rightarrow \cdots \rightarrow (\omega, 2, 1, 2)$

$\rightarrow \cdots \rightarrow (\omega, 3, 1, 4)$

$\rightarrow \cdots \rightarrow (\omega, 4, 1, 6)$

$\cdots$

Replace $(\omega, 2, 1, 2)$ by $(\omega, \omega, 1, \omega)$



Forward Algorithms

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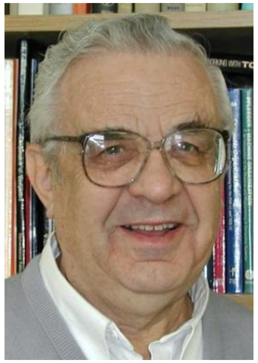
$\cdots$

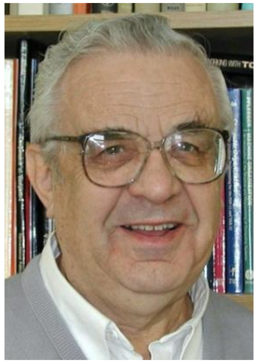
Replace $(\omega, 2, 1, 2)$ by $(\omega, \omega, 1, \omega)$

Observe: the replacement is „safe“ with respect to coverability, all configurations under $(\omega, \omega, 1, \omega)$ are coverable



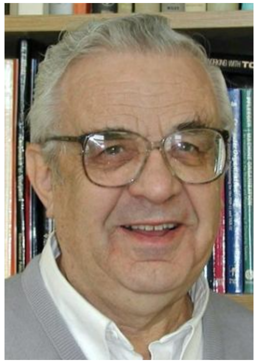
Forward Algorithms

- Theorem (Karp, Miller 69): The Karp-Miller graph is always finite.



Forward Algorithms

- Theorem (Karp, Miller 69): The Karp-Miller graph is always finite.
- Theorem (Karp, Miller 69): A state is coverable iff it is covered by some node of the Karp-Miller graph.



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Forward Algorithms

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- Theorem (Mayr, Meyer 81): The Karp-Miller graph can have non-primitive recursive size.
- So forward-search more expensive than backward-search?

Forward Algorithms

- Expand, Enlarge, Check (Geeraerts *et al*. 2004)
  - The Karp-Miller acceleration is „exact“ with respect to coverability: It only introduces an ω when it is safe to do so.
  - Construct instead a sequence of „underapproximations“ and „overapproximations“ :
    - the i-th underapproximation contains the state spaces of all instances with at most i processes.
    - the i-th overapproximation identifies „more than i processes“ with „arbitrarily many“.

Forward Algorithms

- Theorem (Geeraerts *et al*. 2004): The Expand-Enlarge-Check algorithm terminates.
  - If q_d is coverable, then some underapproximation discovers it.
  - If q_d is not coverable, then let K be the largest number (not ω!) in the (finite) Karp-Miller graph. The overapproximation for $i = K$ is at least as precise as the Karp-Miller graph.

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- Theorem [Majumdar, Zhang 2013]: The EEC algorithm solves coverability in exponential space.

The Leaderless Case

- Karp-Miller graph for a leaderless parametrized configuration:
  - Initial ω-configuration of the form $(\omega, \ldots, \omega, 0, \ldots, 0)$

The Leaderless Case

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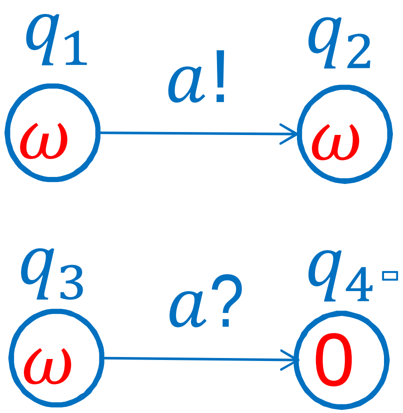

$$(\ldots, \omega, \omega, \omega, 0, \ldots) \xrightarrow{a} (\ldots, \omega, \omega, \omega, 1, \ldots)$$

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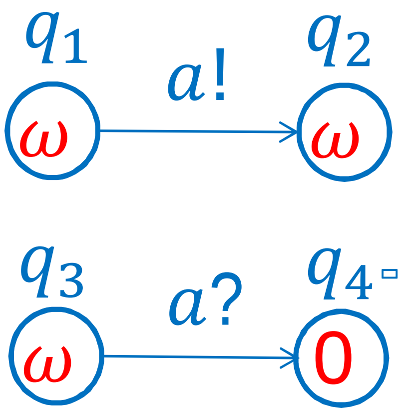

$$(\ldots, \omega, \omega, \omega, 0, \ldots) \xrightarrow{a} (\ldots, \omega, \omega, \omega, 1, \ldots)$$

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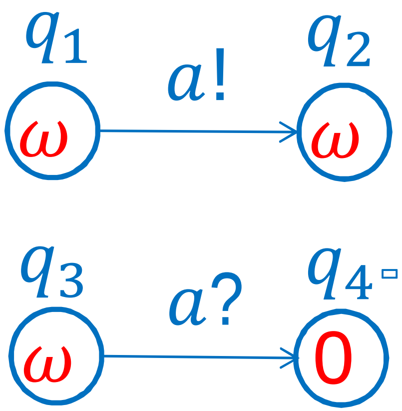

$$(\ldots, \omega, \omega, \omega, 0, \ldots) \xrightarrow{a} (\ldots, \omega, \omega, \omega, 1, \ldots)$$

is accelerated to

$$(\ldots, \omega, \omega, \omega, 0, \ldots) \xrightarrow{a} (\ldots, \omega, \omega, \omega, \omega, \ldots)$$

  - So every ω-configuration of the graph contains only 0´s and ω´s.

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Consequence: The graph has at most 2^n states

Fact 2: Along every path the ω's increase monotonically

Consequence: Every simple path of the graph has at most length n, and so the coverability problem is in NP

The Leaderless Case

Fact 3: If

$$\begin{tikzcd}
 & (\dots, 0, \omega, 0, \dots) \arrow[dl, "a"'] \arrow[dr, "b"] & \\
(\dots, \omega, \omega, 0, \dots) & & (\dots, 0, \omega, \omega, \dots)
\end{tikzcd}$$

then

$$\begin{tikzcd}
 & (\dots, 0, \omega, 0, \dots) \arrow[dl, "a"'] \arrow[dr, "b"] & \\
(\dots, \omega, \omega, 0, \dots) \arrow[dr, "b"'] & & (\dots, 0, \omega, \omega, \dots) \arrow[dl, "a"] \\
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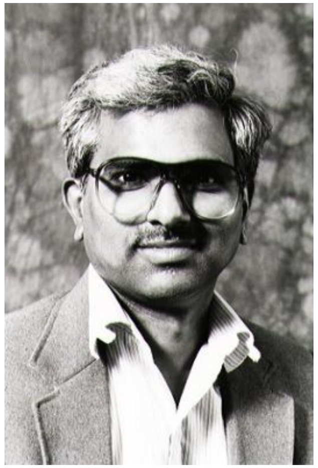
(\ldots, \omega, \omega, 0, \ldots) \arrow[dr, "b"'] & & (\ldots, 0, \omega, \omega, \ldots) \arrow[dl, "a"] \\
 & (\ldots, \omega, \omega, \omega, \ldots) &
\end{tikzcd}
$$

Consequence:
We can compute the set of ω's reachable after 1,2, … n steps in PTIME

The Leaderless Case

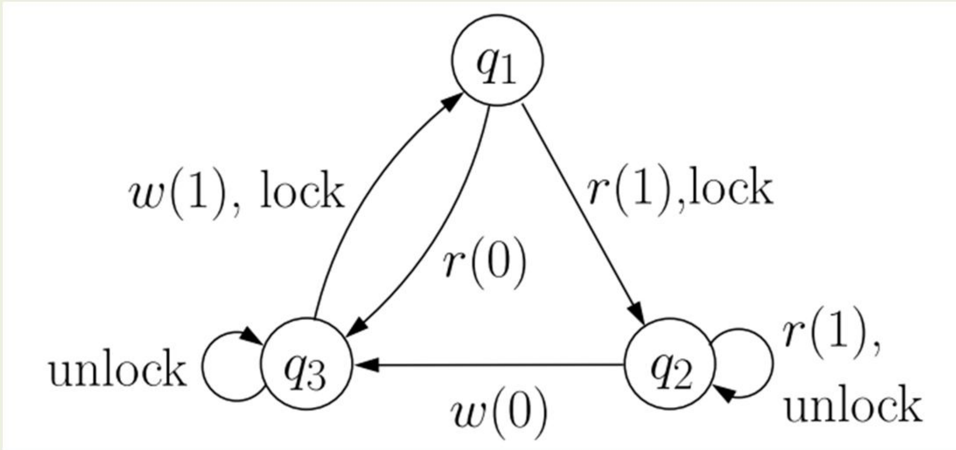
Theorem (German, Sistla *92*): The coverability problem for leaderless parametrized configurations is solvable in PTIME.



Comm. Mechanisms: Shared memory

Shared memory

- A lock for every shared variable
- Process owning the lock can perform reads and writes



Examples

– Multithreaded programs

Shared memory

- Shared memory communication can be simulated by rendez-vous communication, and vice versa.

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  - Rendez-vous → Shared memory: one memory value per action, plus one for „currently empty“
- Theorem: The coverability problem for shared-memory systems is EXPSPACE-complete

The Leaderless Case

- However: the simulation does not preserve „leaderlessness“

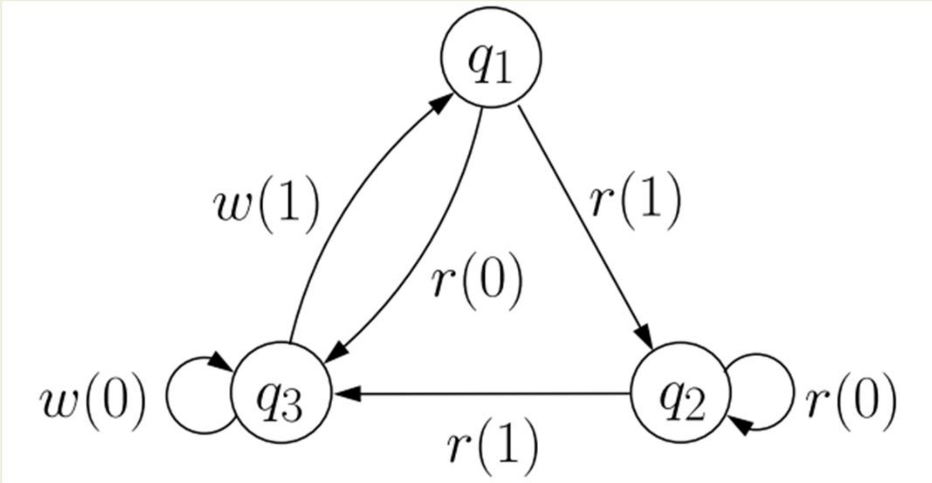
Theorem: The coverability problem for shared-memory systems EXPSPACE-complete for leaderless initial configurations

Comm.Mech: Lock-free Shared Memory

Lock-free shared memory

– Concurrent reads and writes allowed

– Interleaving semantics



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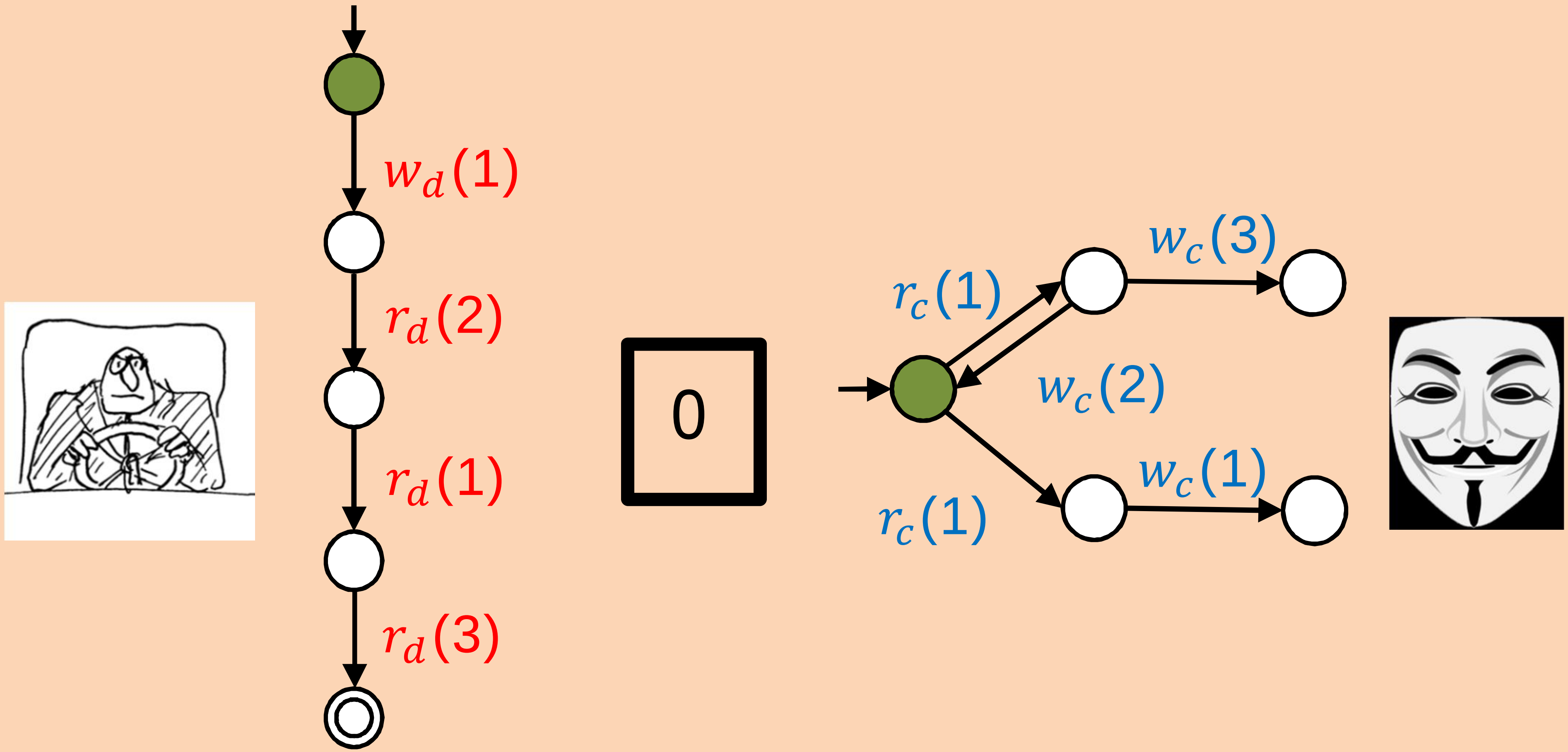

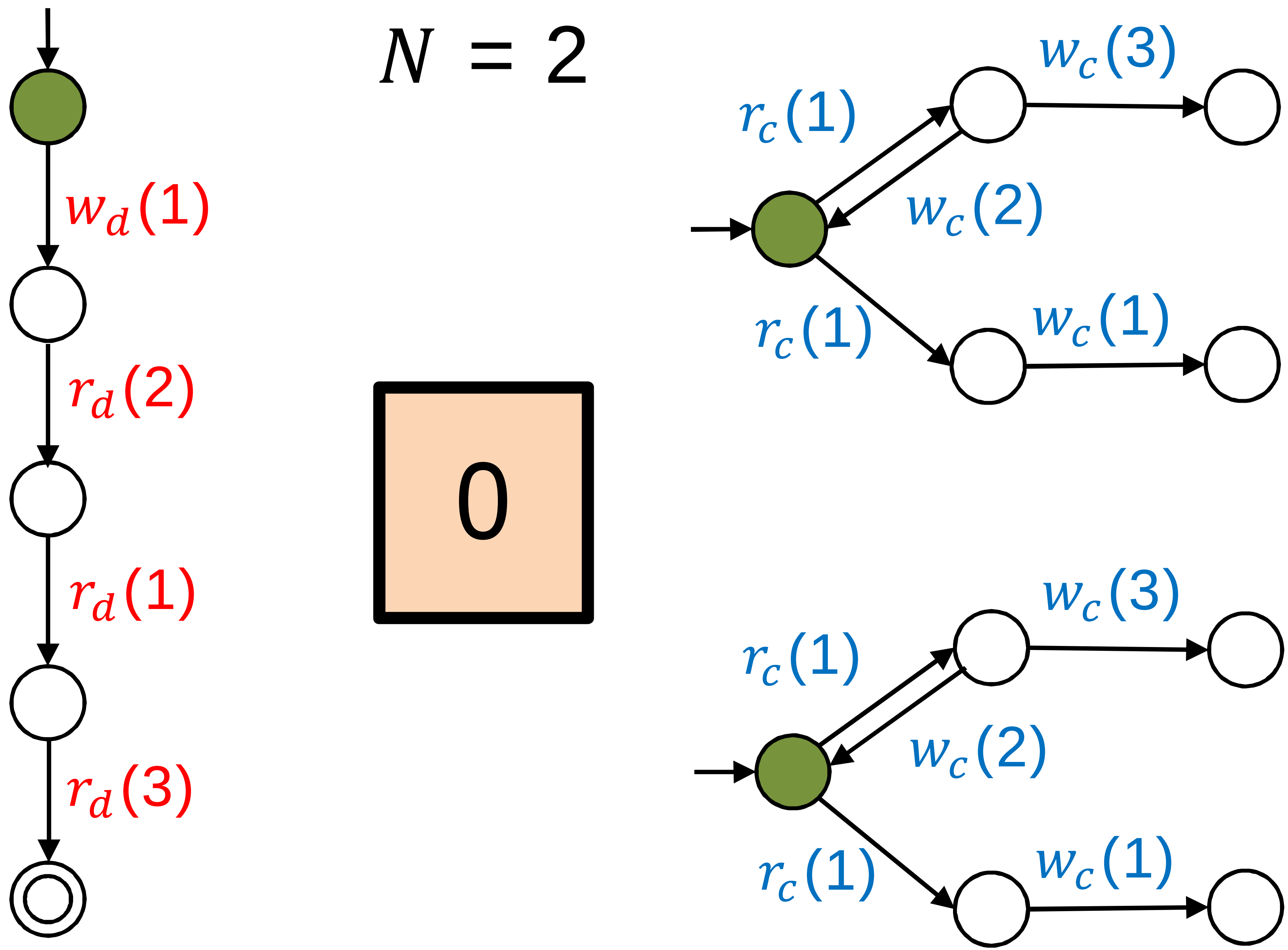

$N = 2$
$w_d(1)$
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$r_d(1)$
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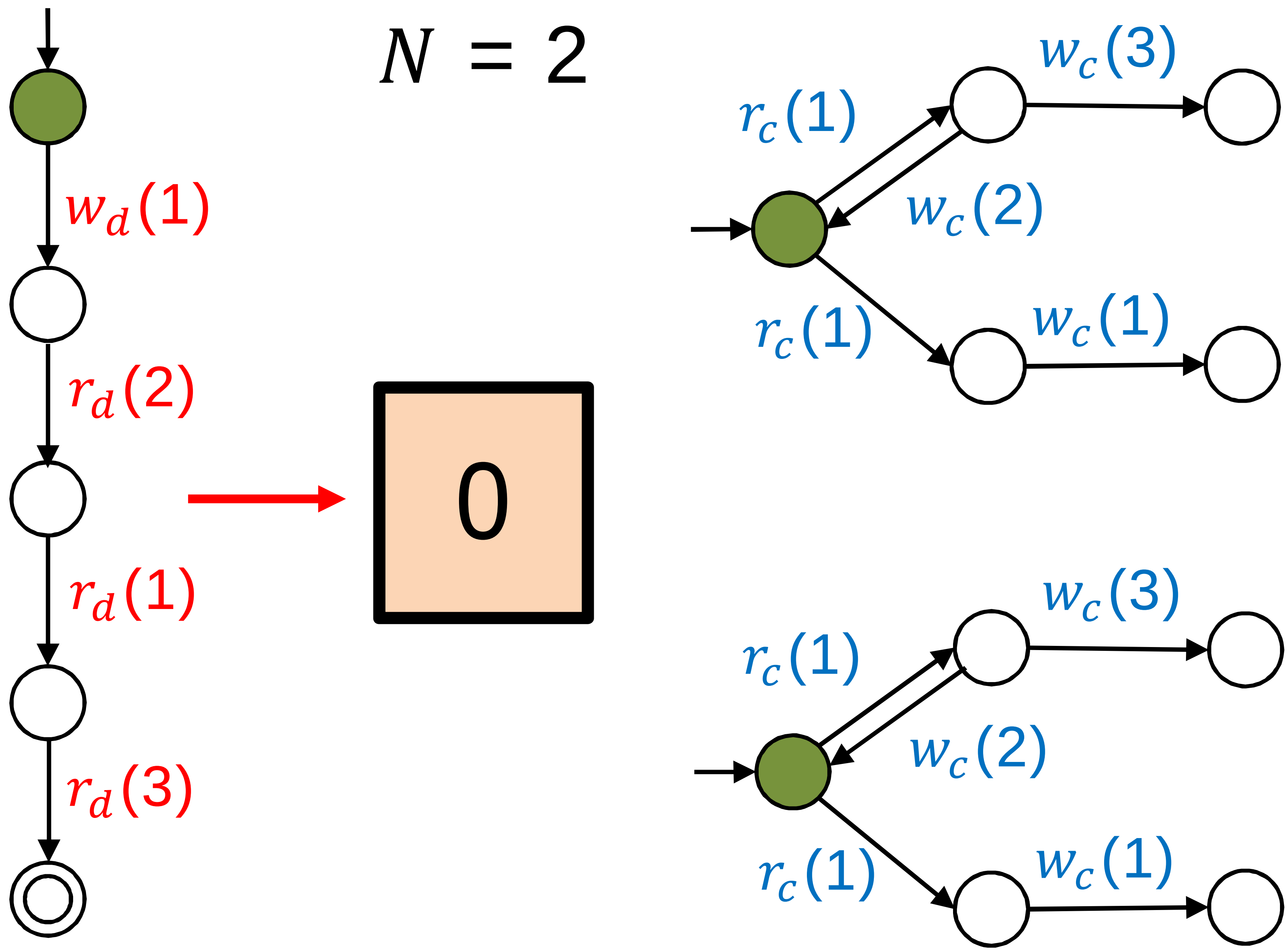

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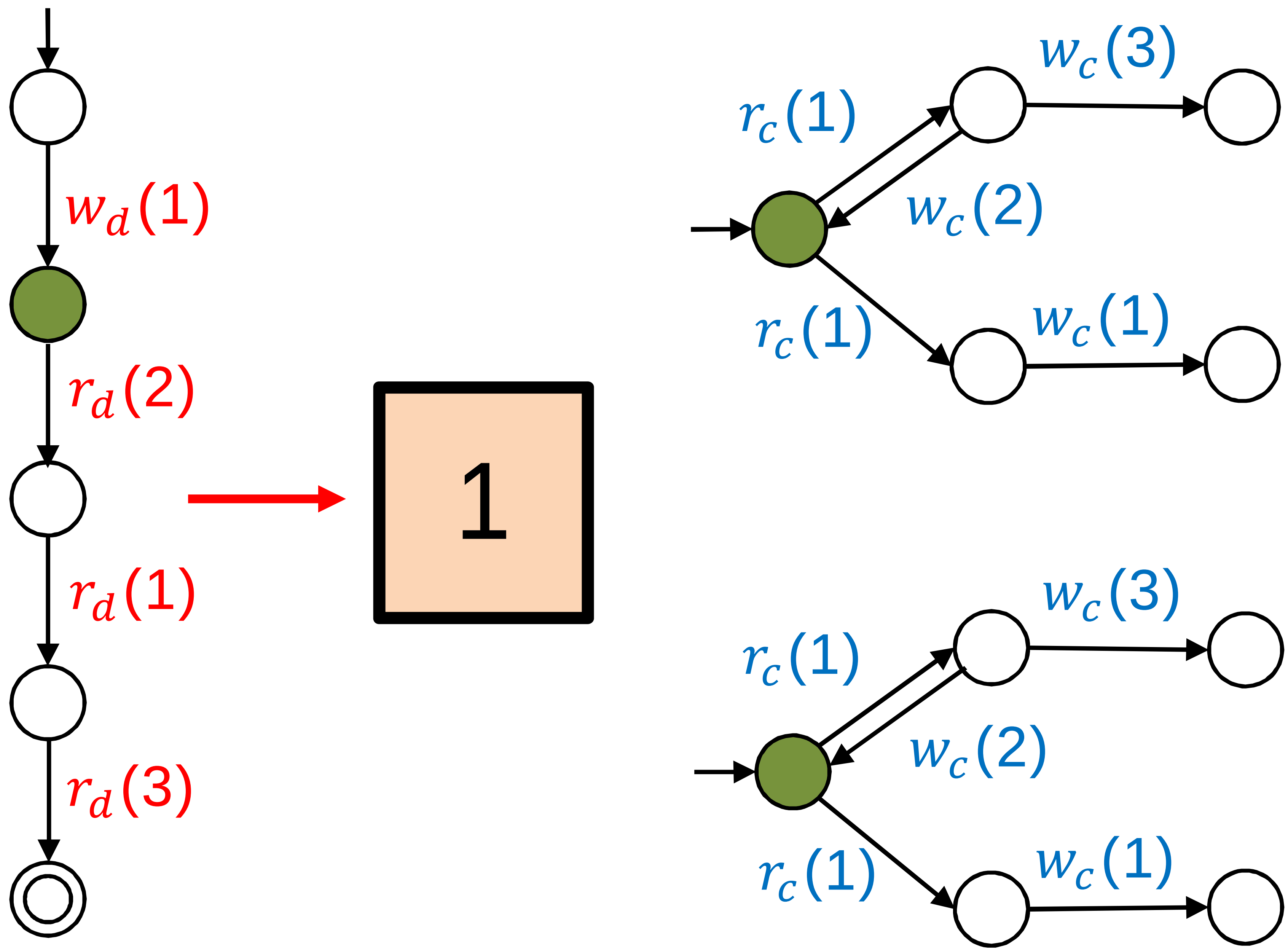

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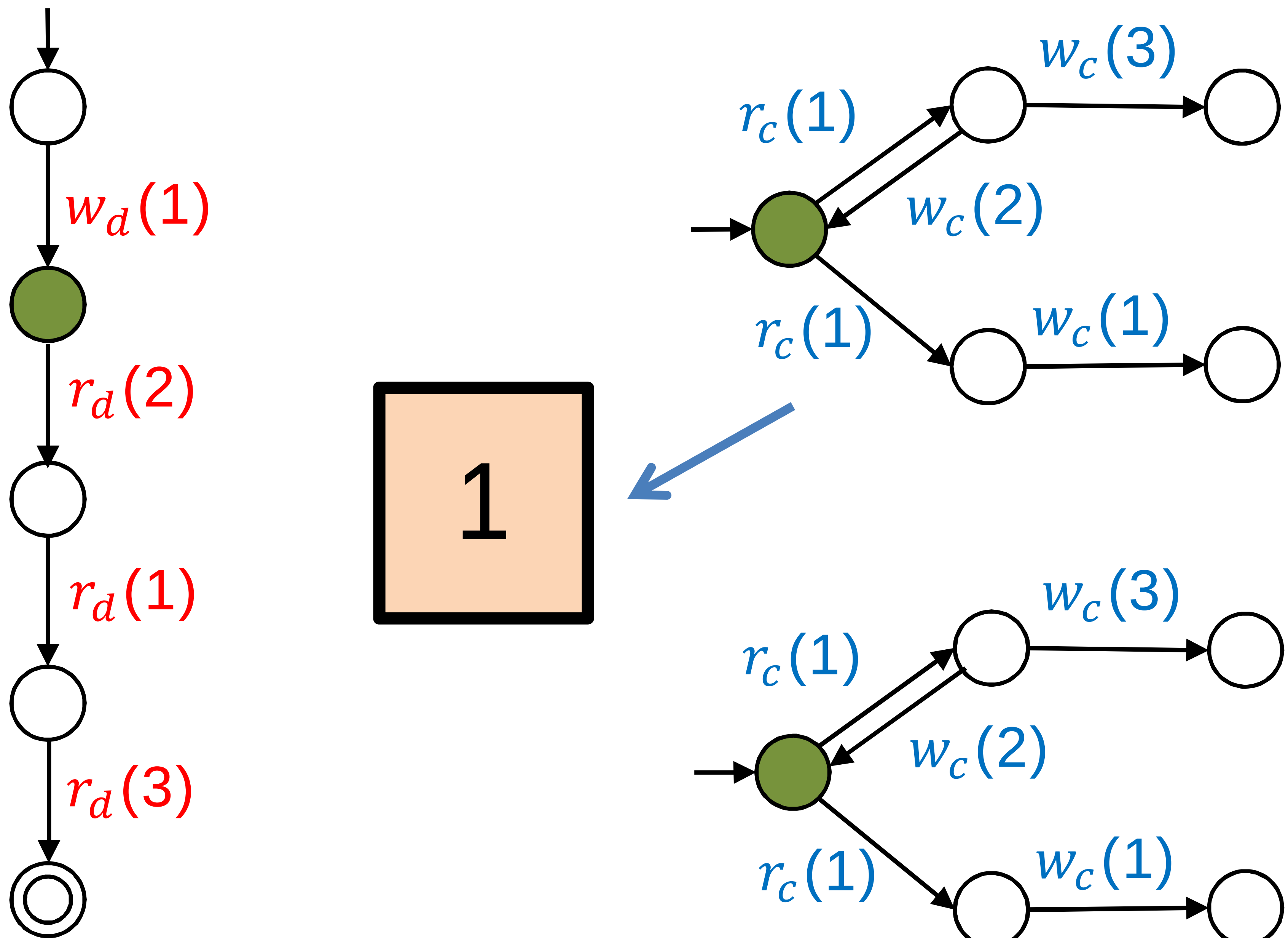

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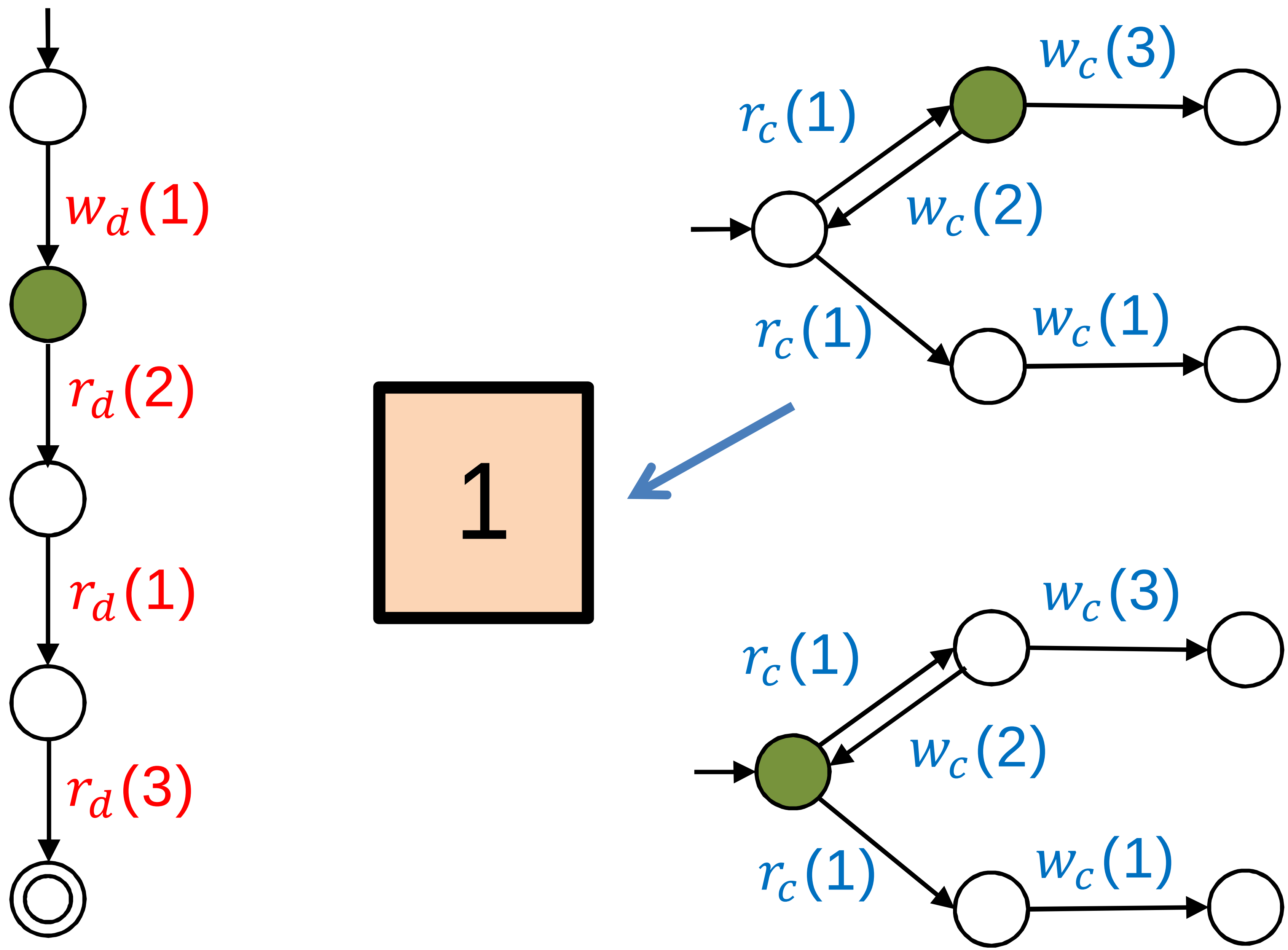

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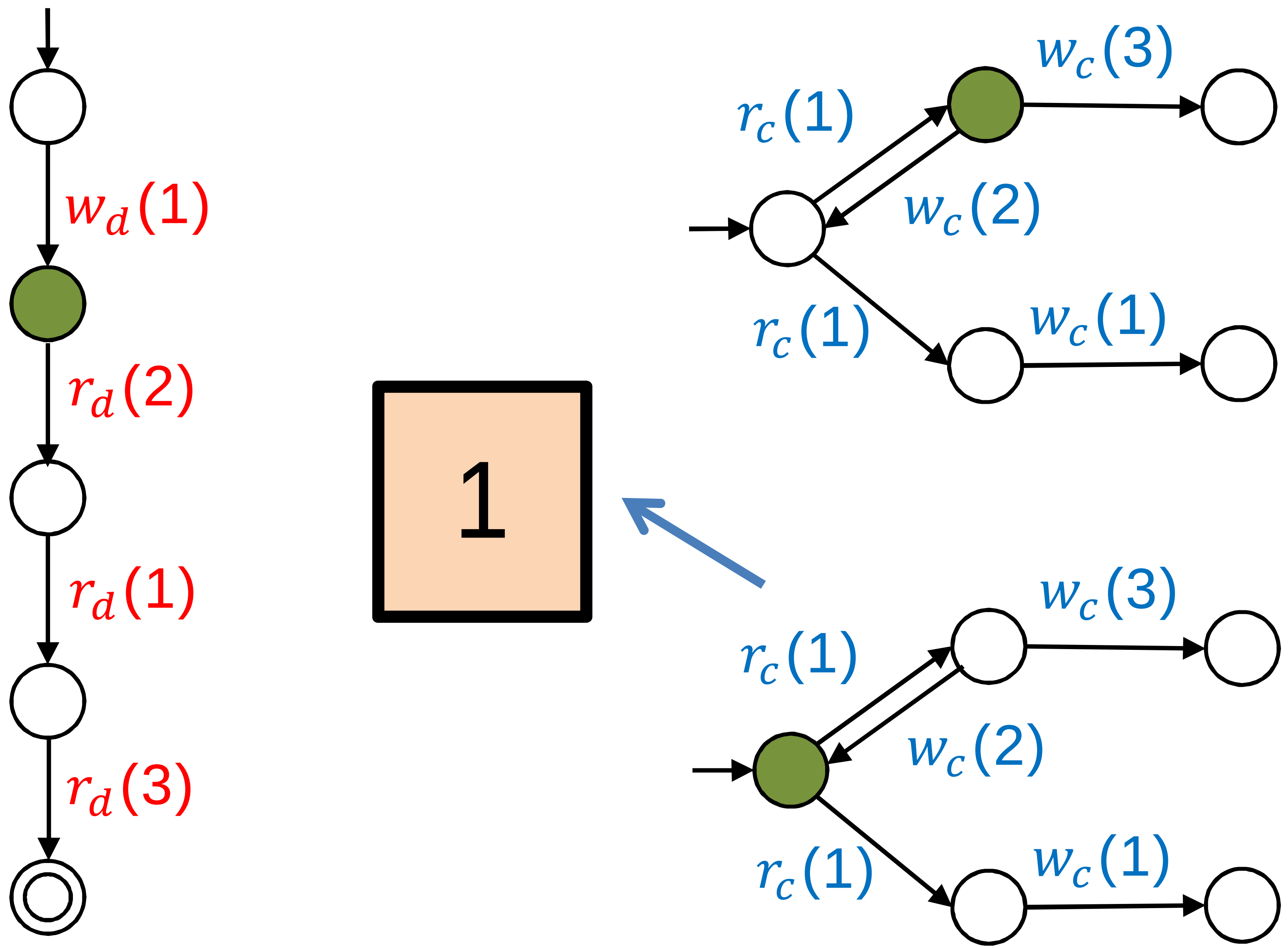

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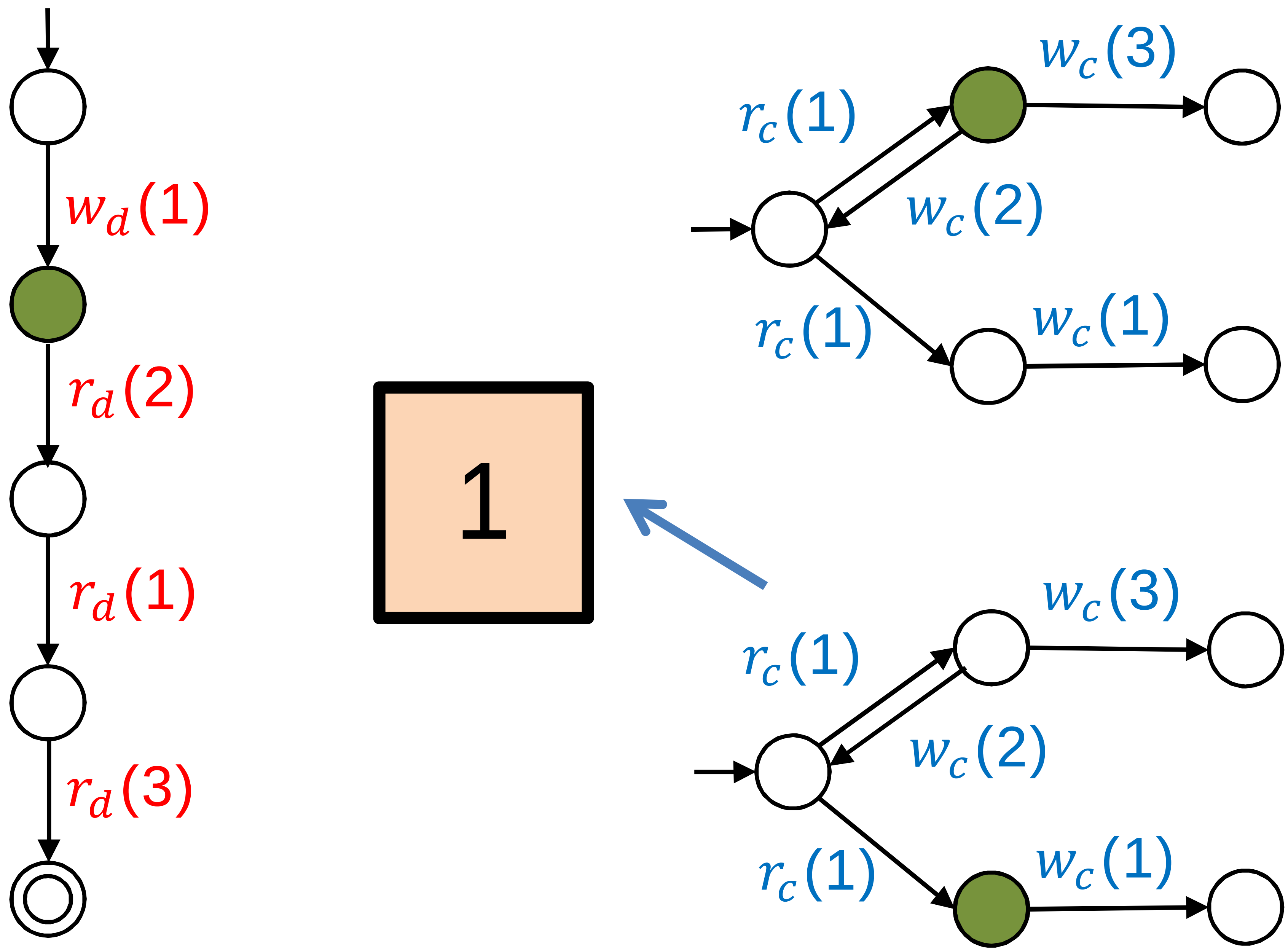

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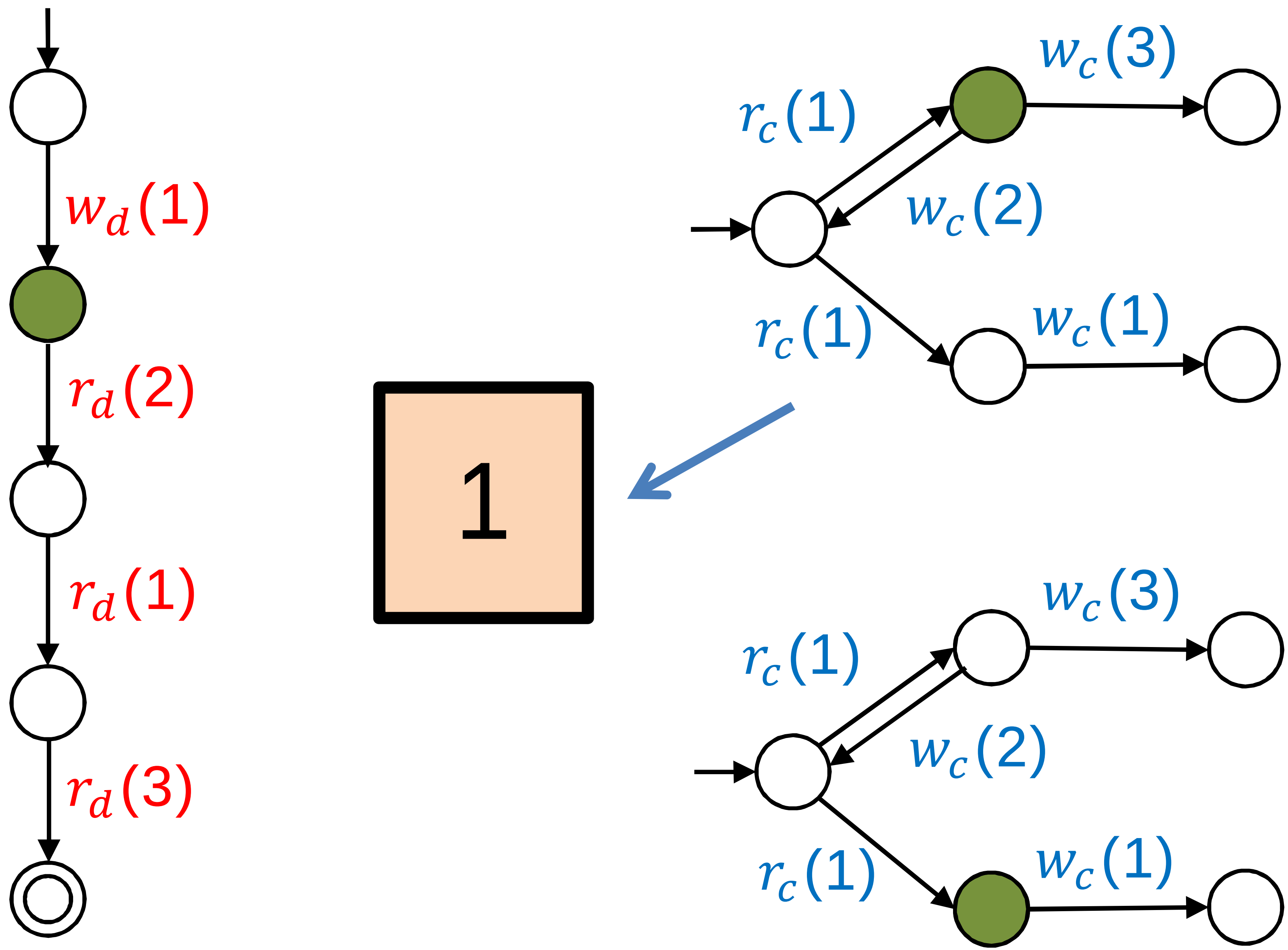

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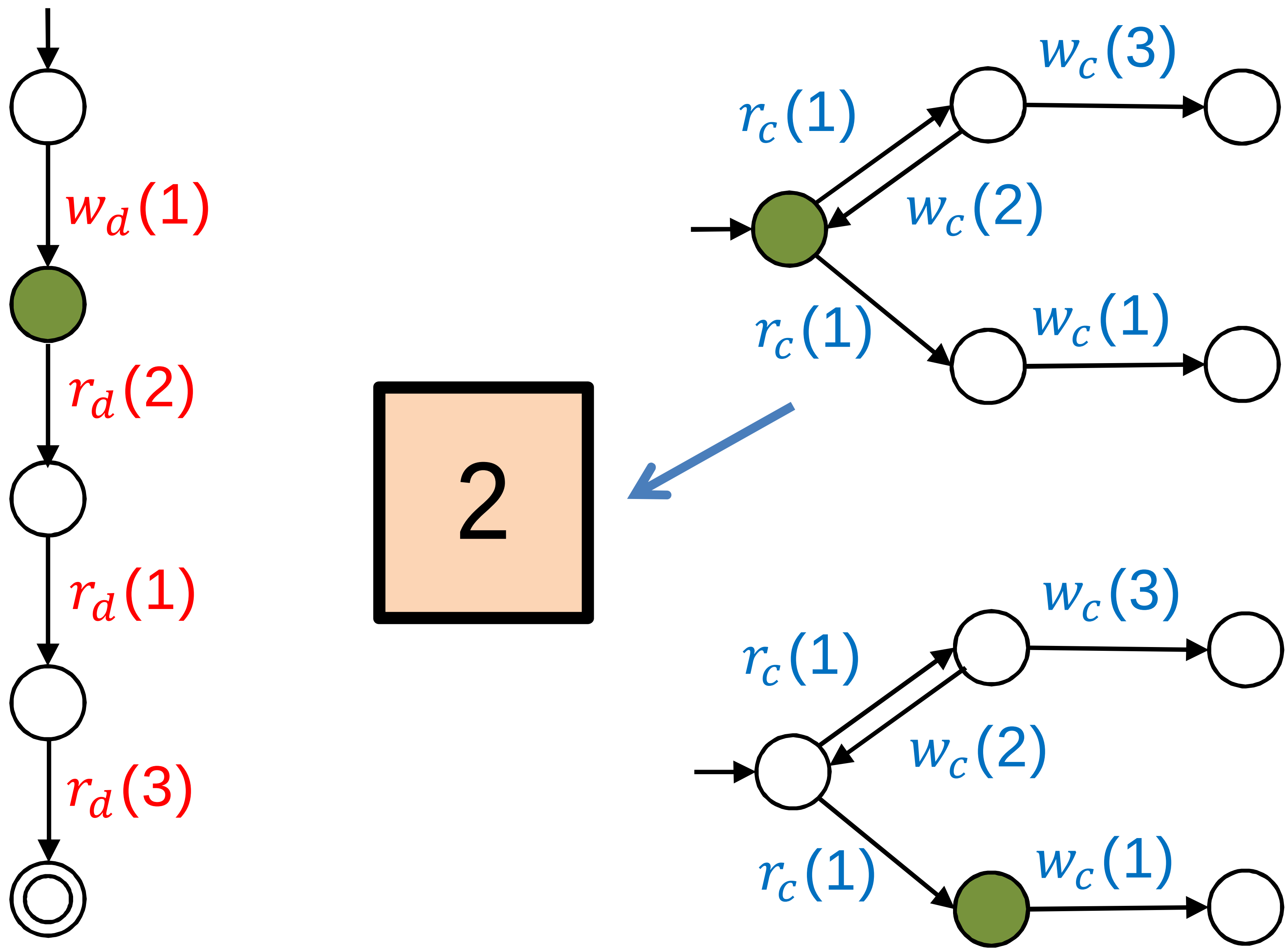

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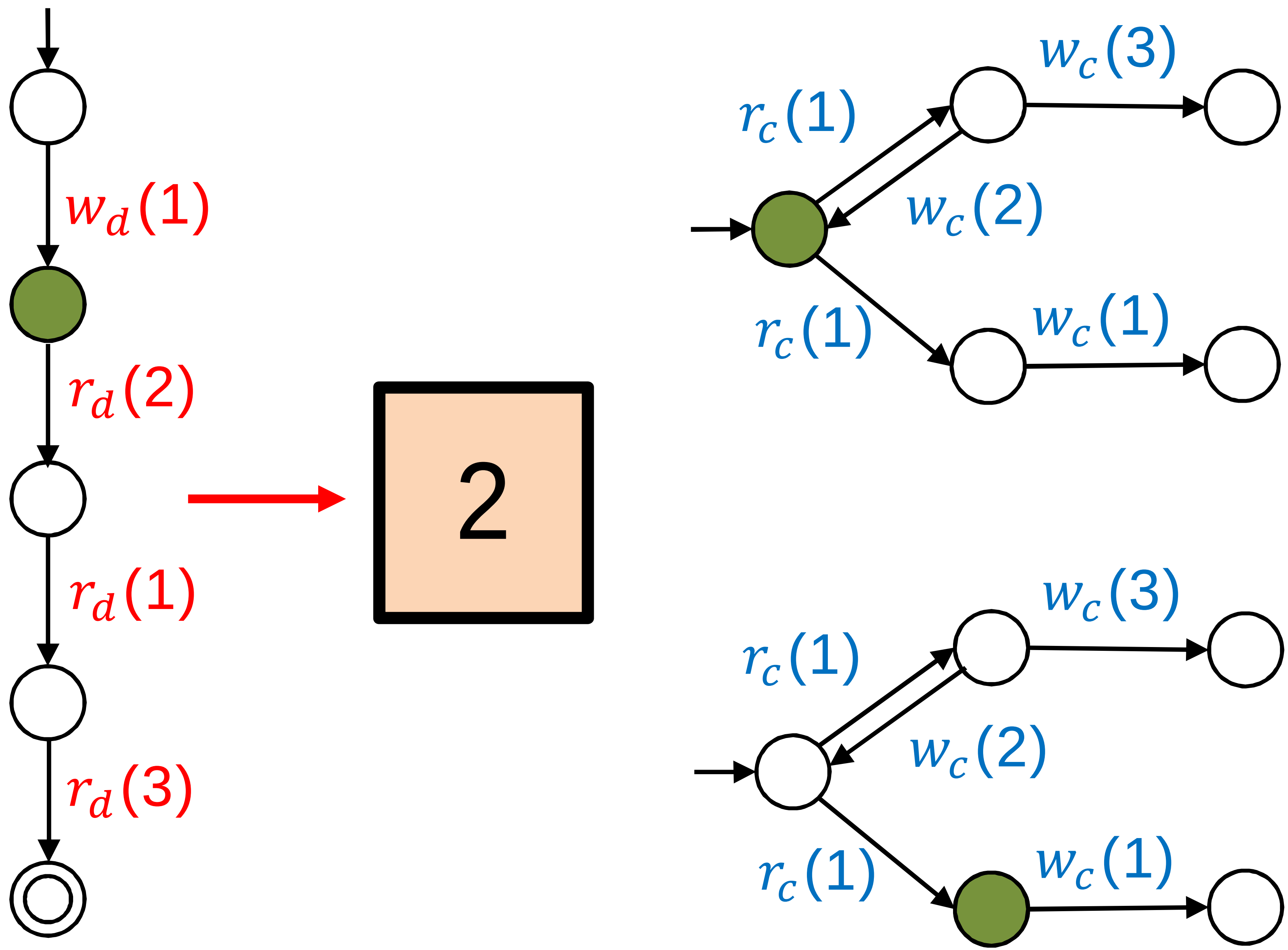

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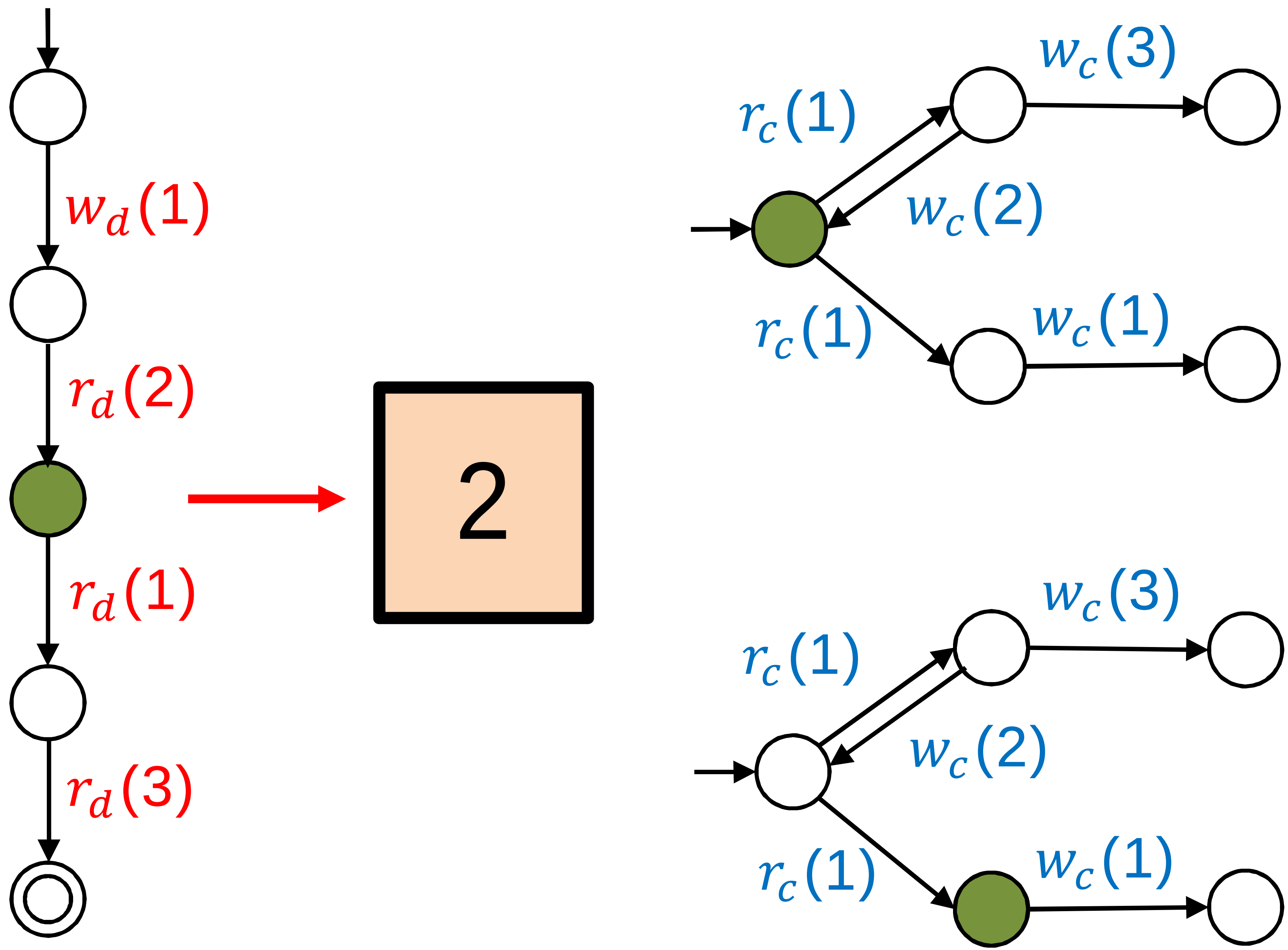
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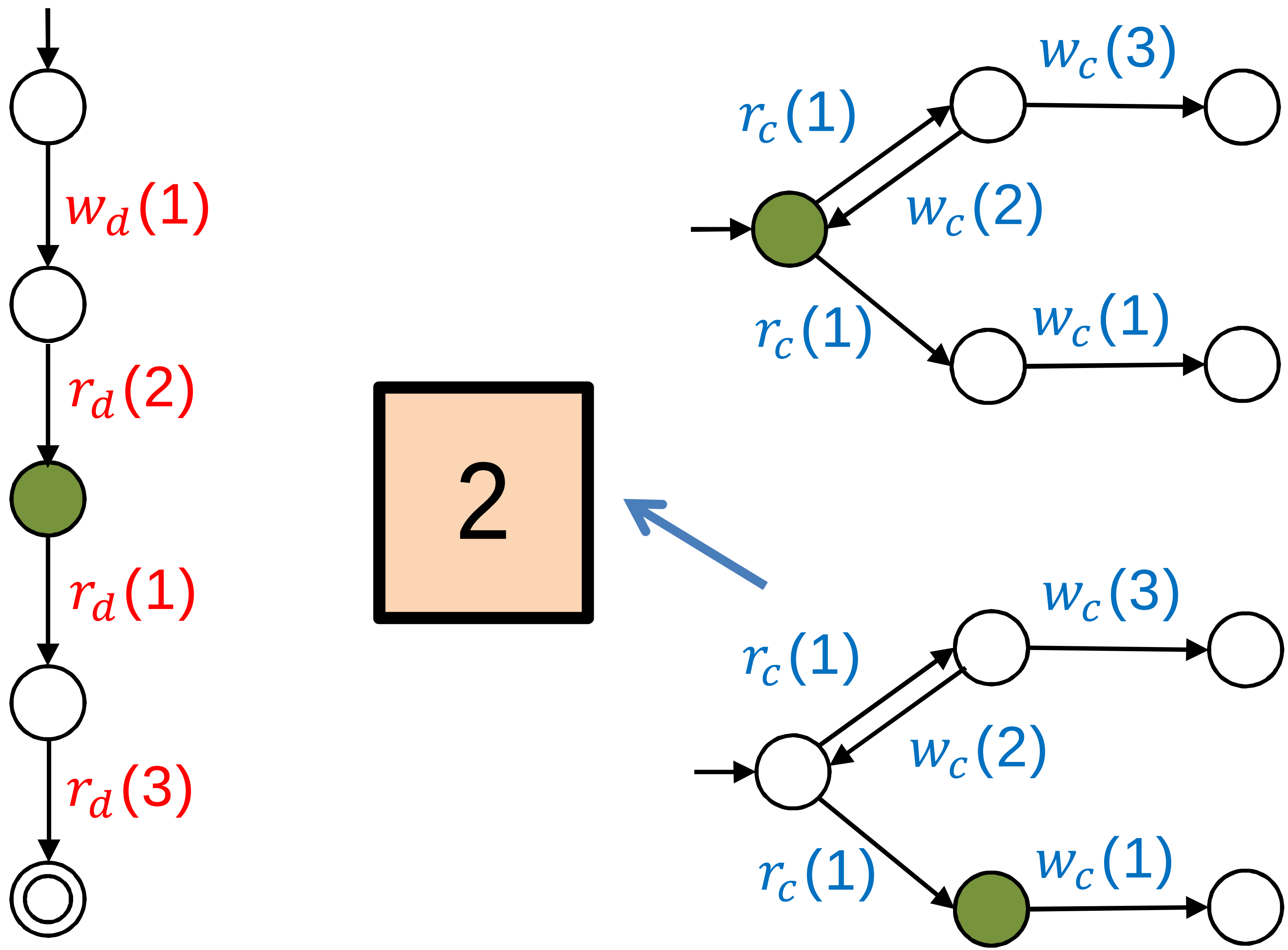

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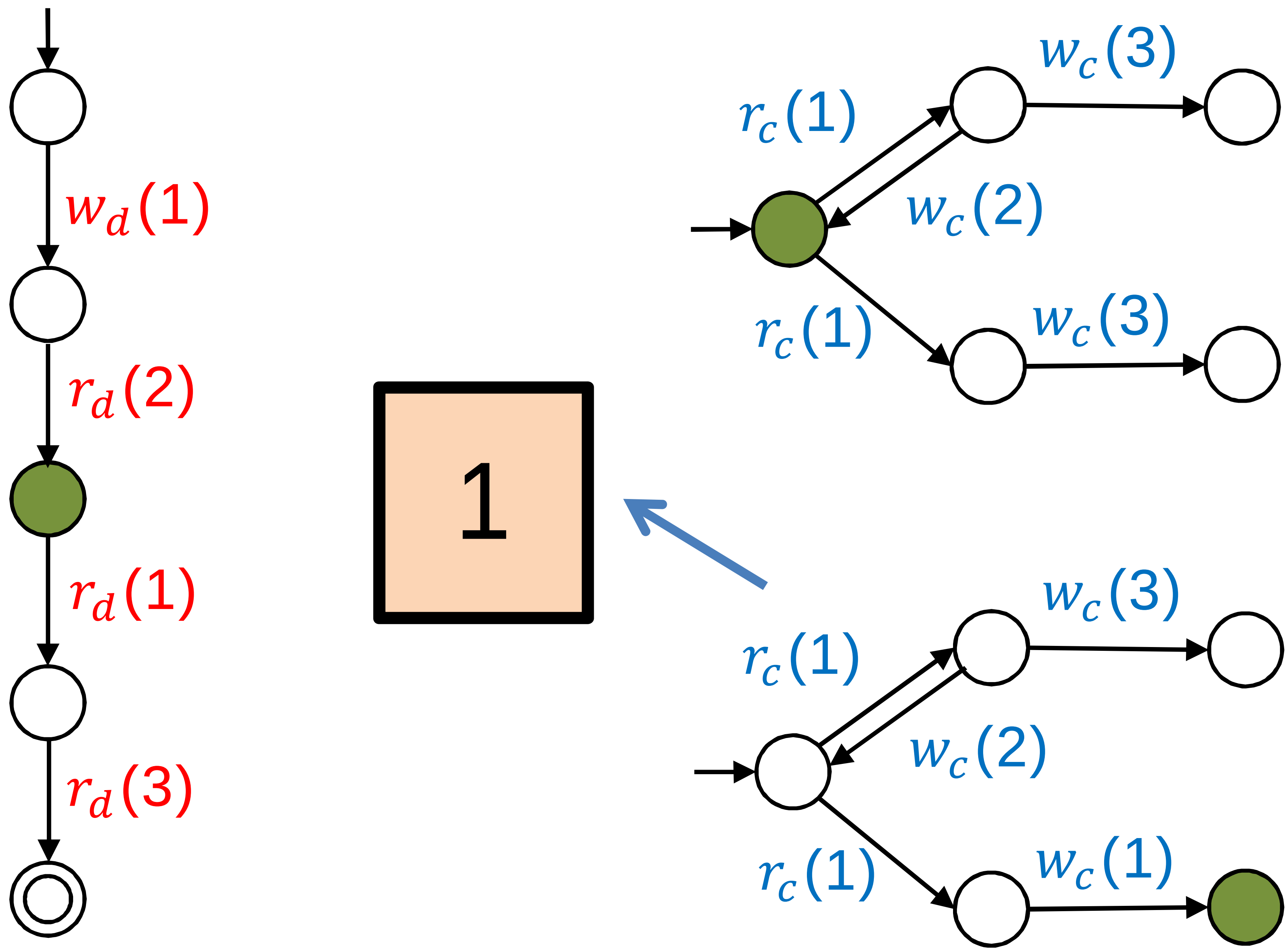

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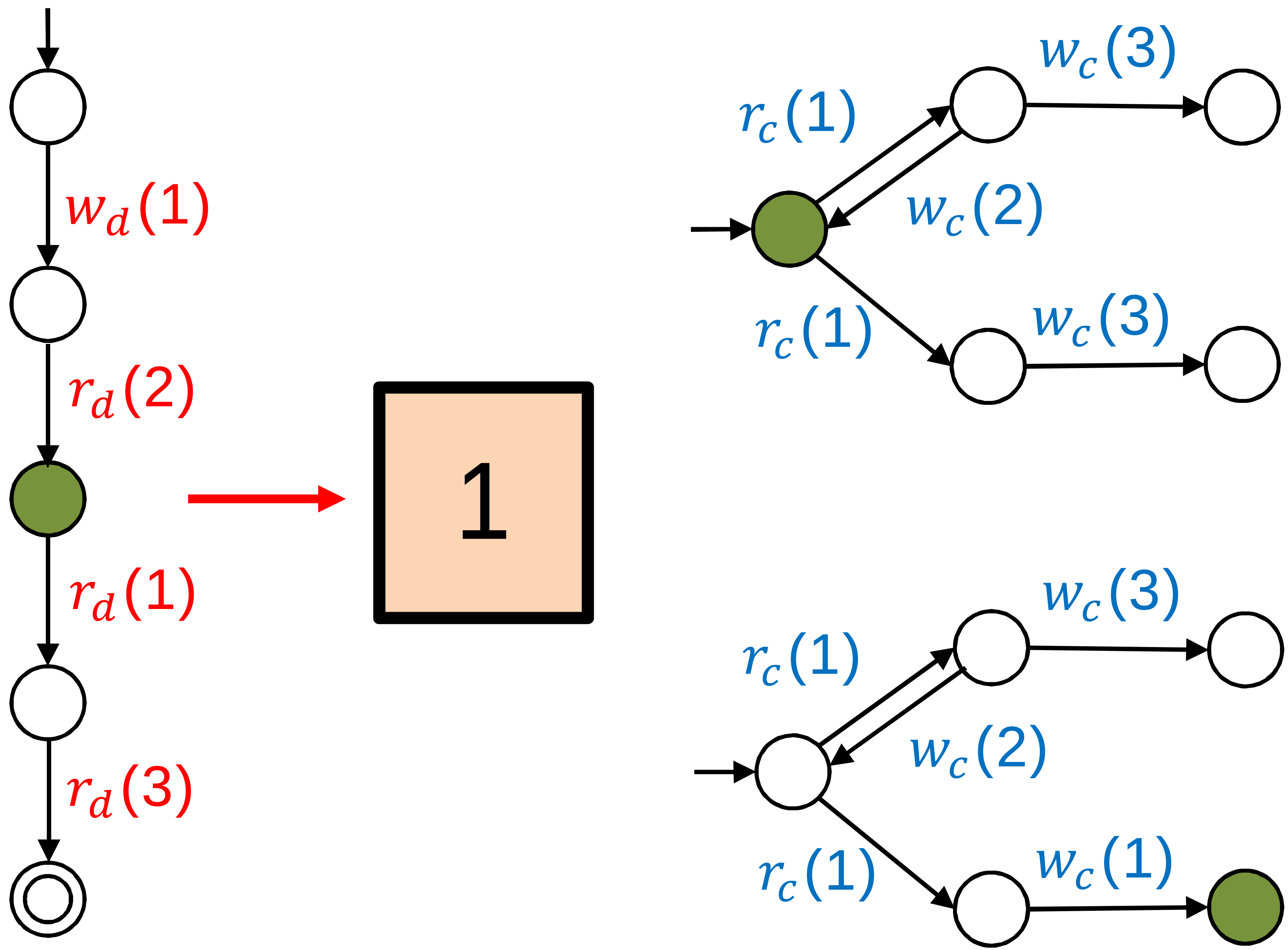

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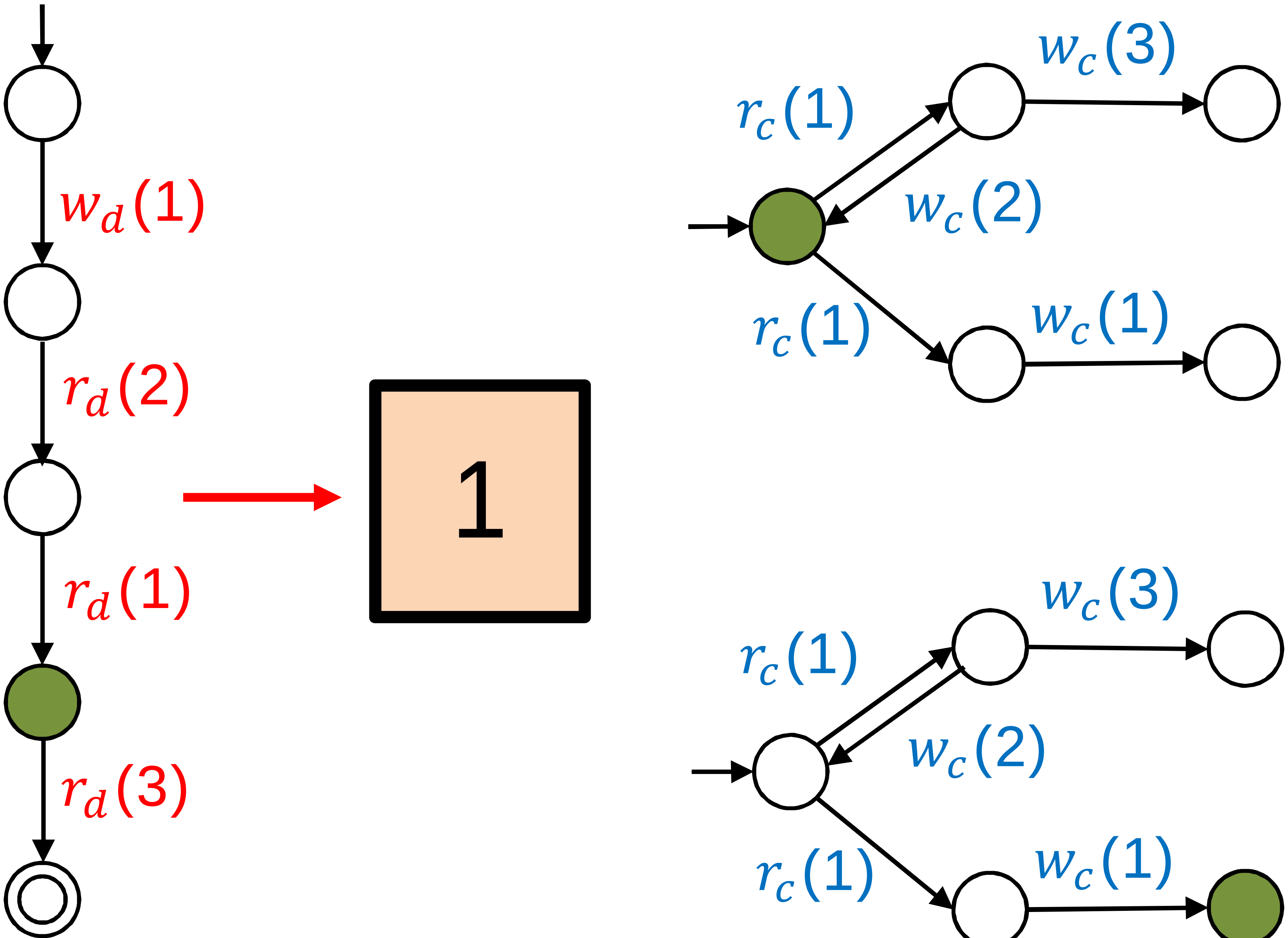

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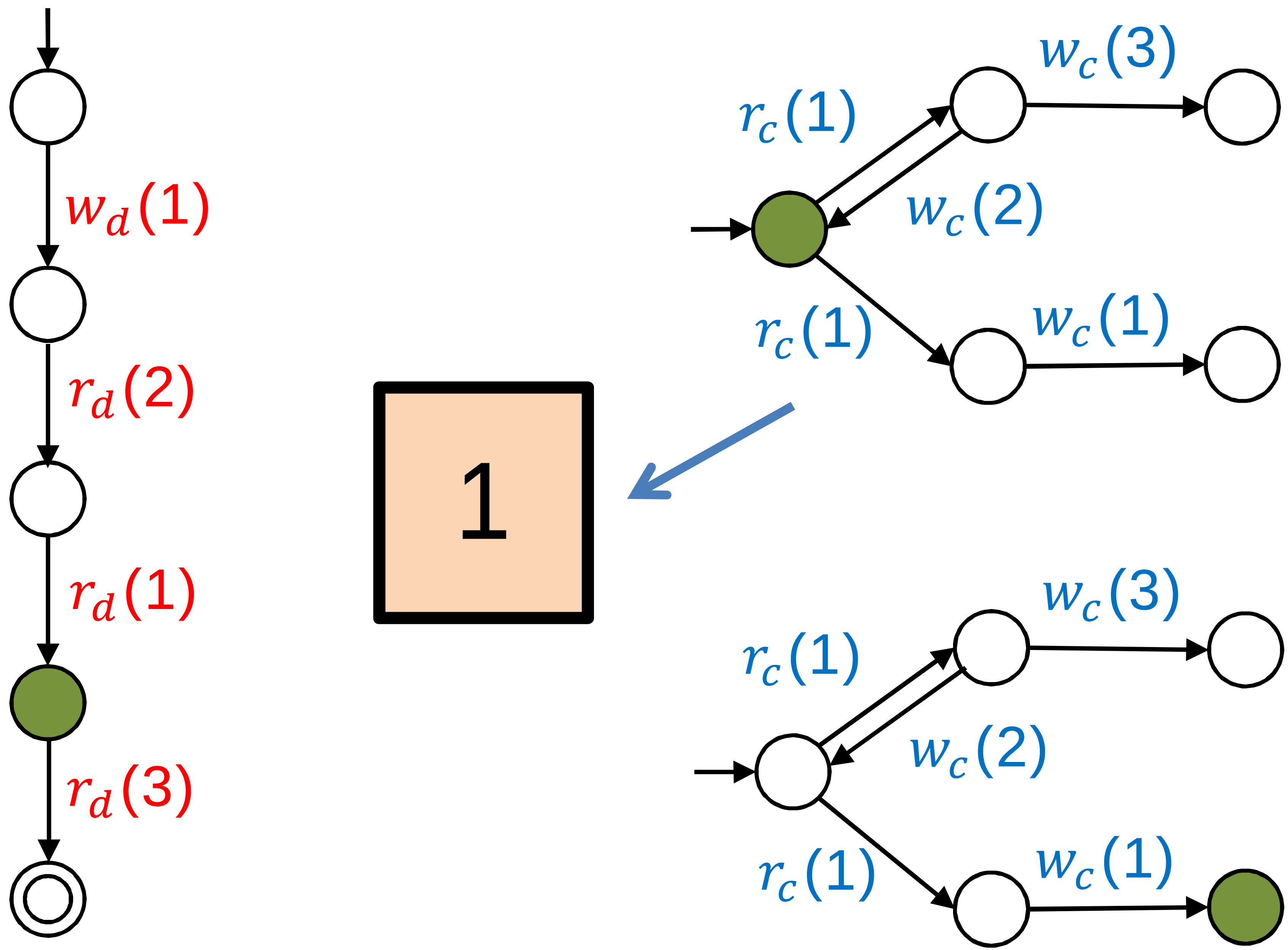

$w_d(1)$
$r_d(2)$
$r_d(1)$
$r_d(3)$
1
$r_c(1)$
$w_c(3)$
$w_c(2)$
$r_c(1)$
$w_c(1)$
$r_c(1)$
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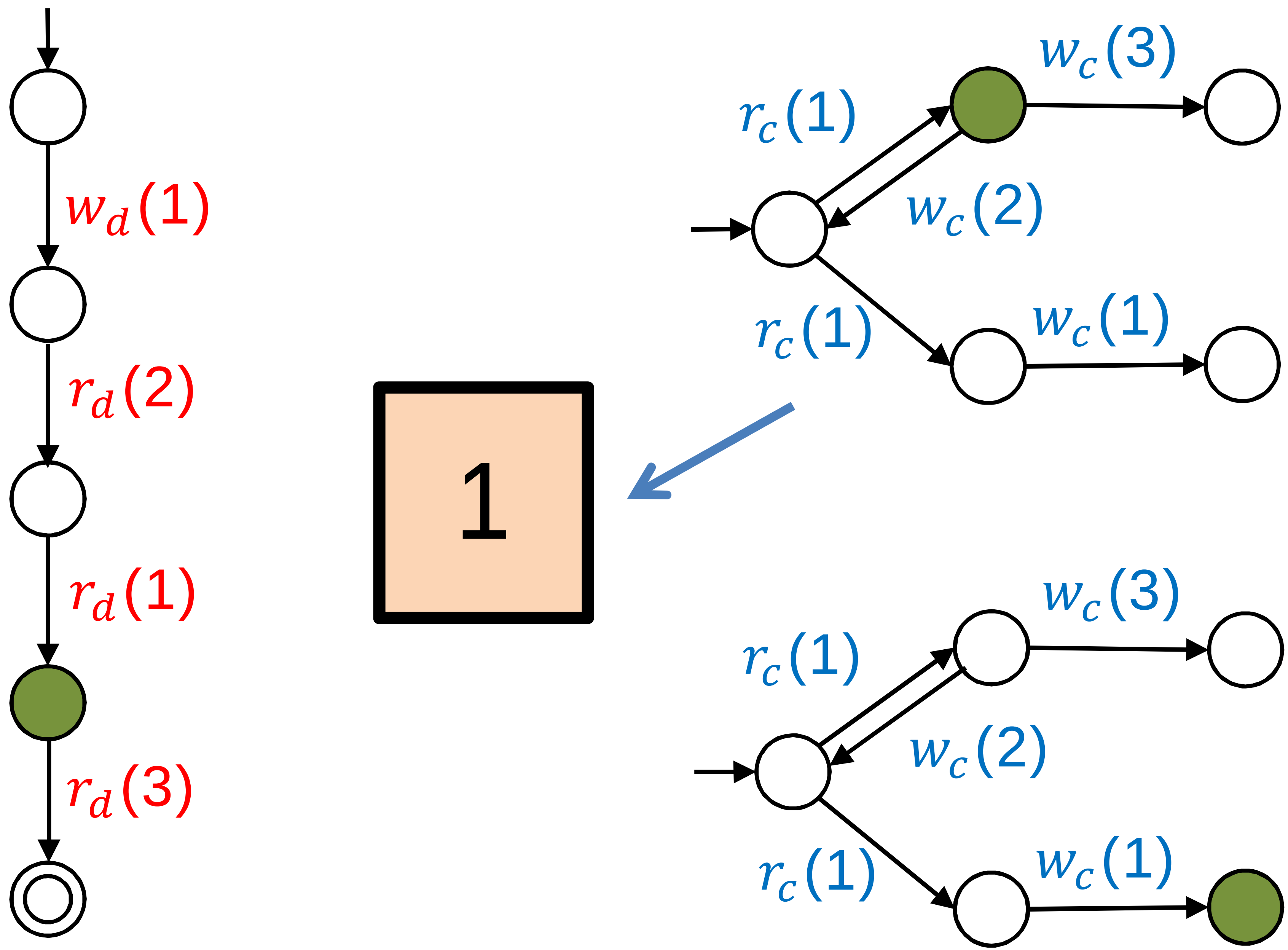
$r_c(1)$
$w_c(1)$



$w_d(1)$
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$r_d(1)$
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1
$r_c(1)$
$w_c(3)$
$w_c(2)$
$r_c(1)$
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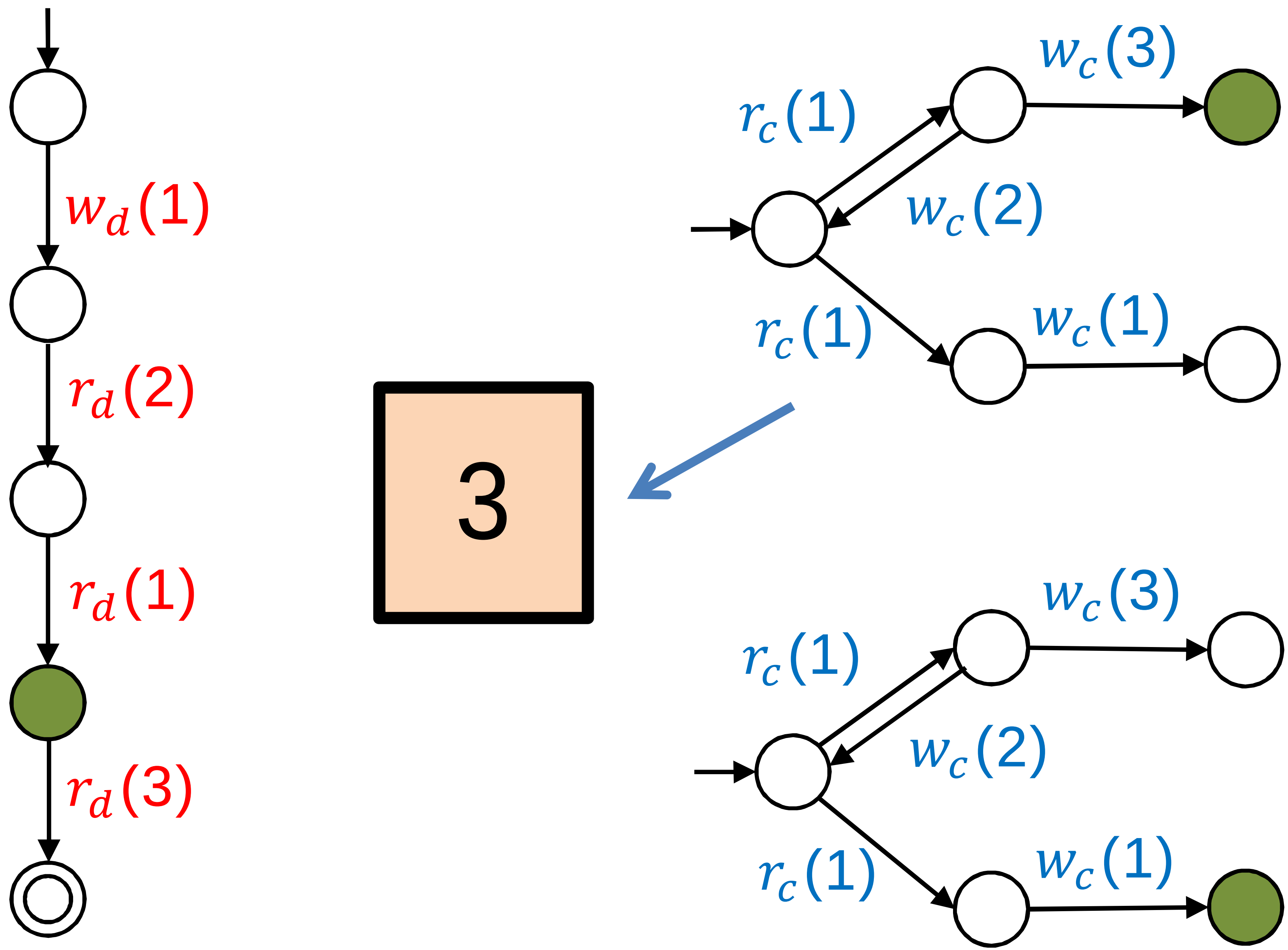
$w_c(3)$
$w_c(2)$
$r_c(1)$
$w_c(1)$



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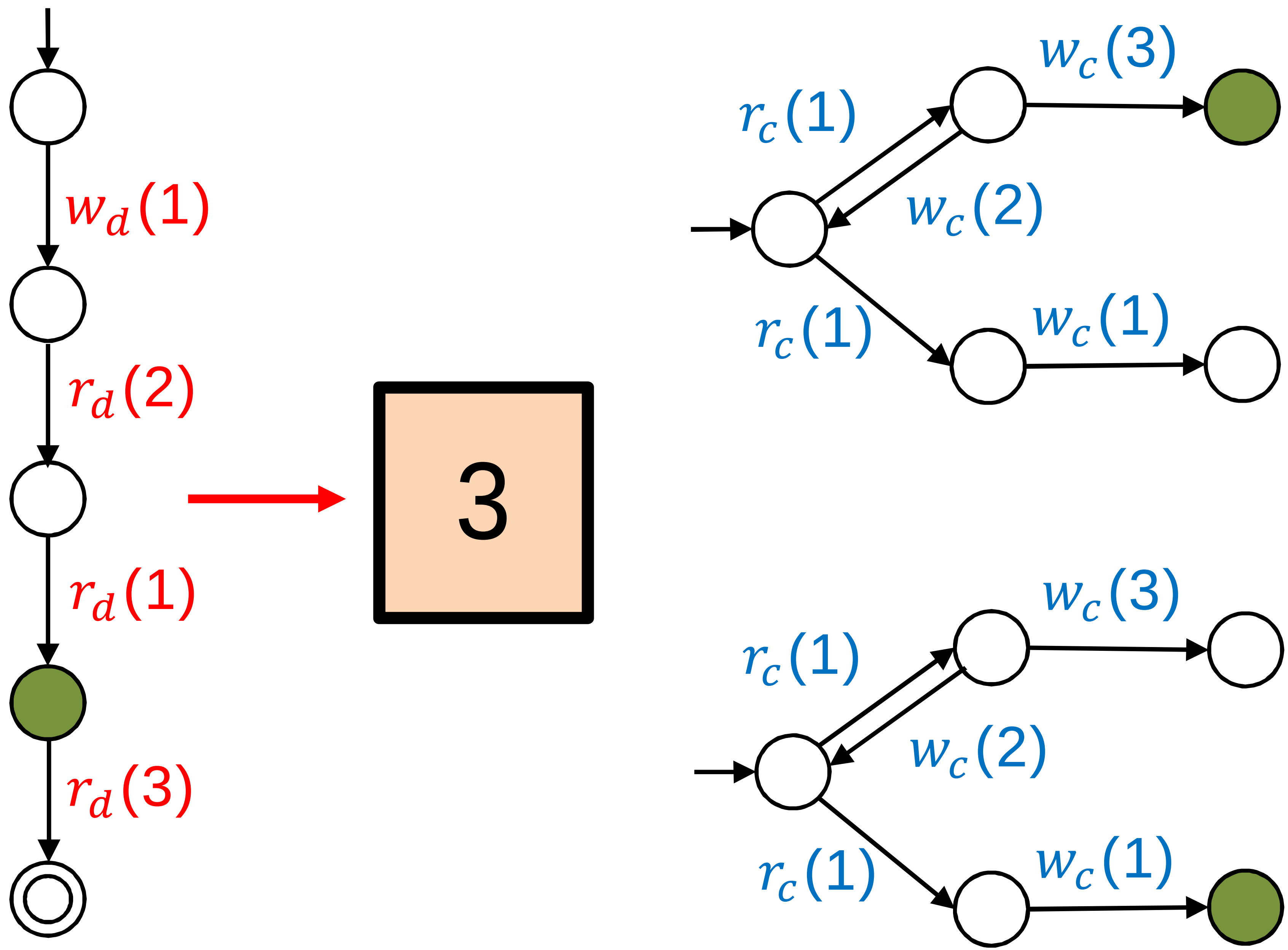
$r_d(1)$
$r_d(3)$
3
$r_c(1)$
$w_c(3)$
$w_c(2)$
$r_c(1)$
$w_c(1)$
$r_c(1)$
$w_c(3)$
$w_c(2)$
$r_c(1)$
$w_c(1)$



$w_d(1)$
$r_d(2)$
$r_d(1)$
$r_d(3)$
3
$r_c(1)$
$w_c(3)$
$w_c(2)$
$r_c(1)$
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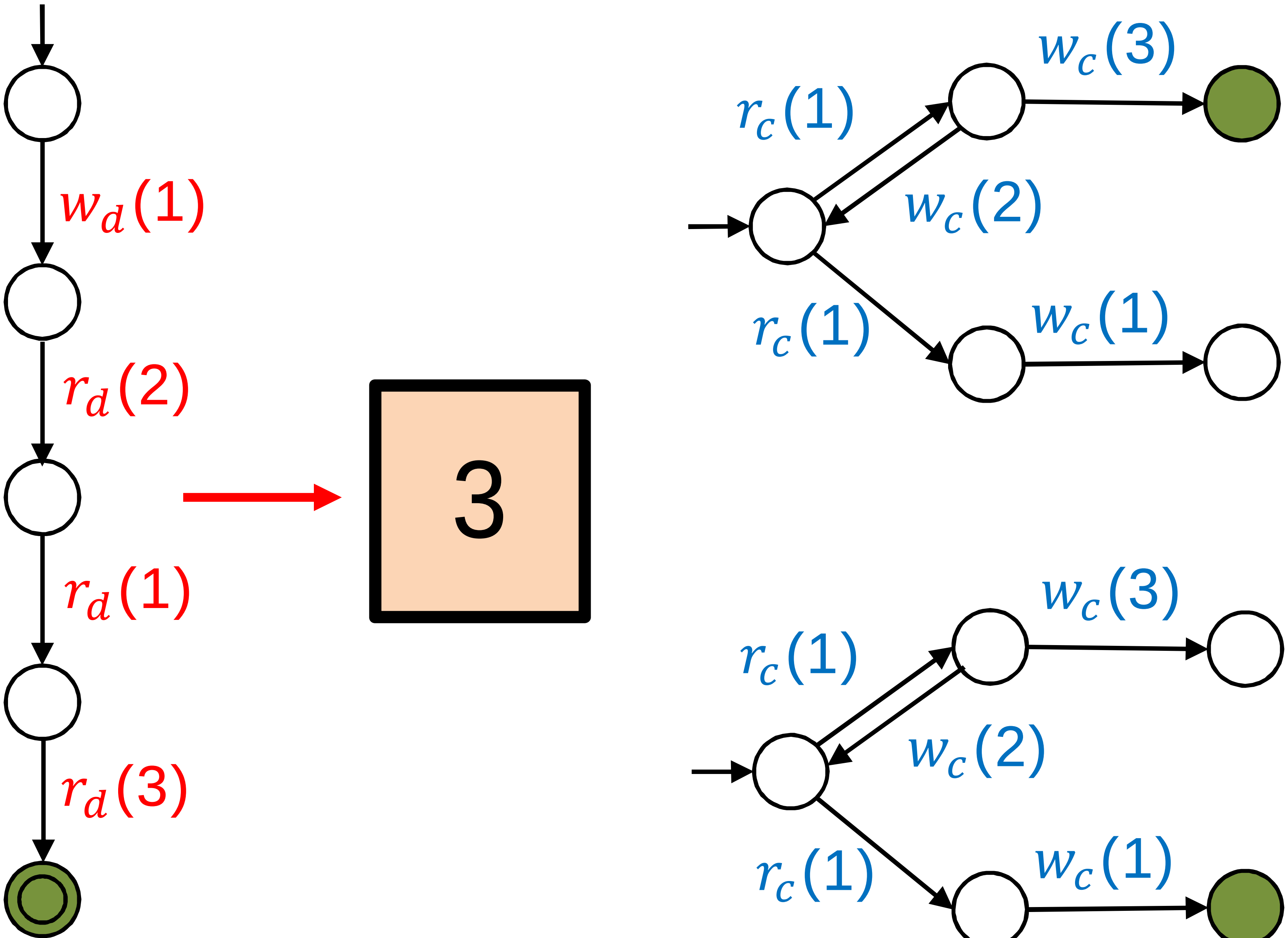
$w_c(1)$



$w_d(1)$
$r_d(2)$
$r_d(1)$
$r_d(3)$
3
$r_c(1)$
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$r_c(1)$
$w_c(3)$
$w_c(2)$
$r_c(1)$
$w_c(1)$

Lock-free Shared Memory

Theorem (E.,Ganty, Majumdar 2013, 2015)

The coverability problem for lock-free shared memory is NP-complete.

In the leaderless case, the problem is polynomial.

Lock-free Shared Memory

- Configuration: triple (q, v, C), where
  - q : state of the leader
  - v : current value of the store
  - C : number of processes in each state of contributor

Lock-free Shared Memory

- We construct the Karp-Miller graph:

If $[q, v, (\omega, 1, 1, 0)] \rightarrow \cdots \rightarrow [q, v, (\omega, 2, 1, 2)]$

then $[q, v, (\omega, 1, 1, 0)] \rightarrow \cdots \rightarrow [q, v, (\omega, 2, 1, 2)]$

$\rightarrow \cdots \rightarrow [q, v, (\omega, 3, 1, 4)]$

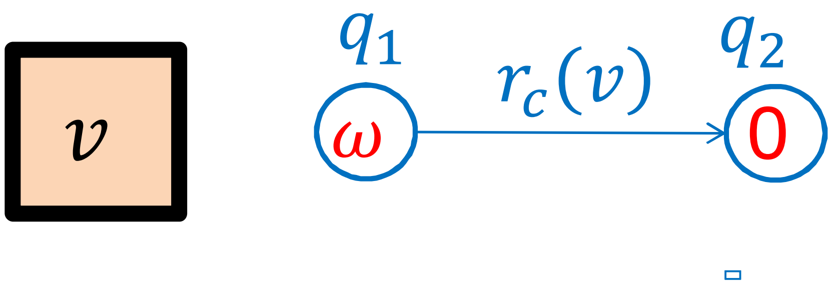
$\rightarrow \cdots \rightarrow [q, v, (\omega, 4, 1, 6)]$

$\cdots$

Replace $[q, v, (\omega, 2, 1, 2)]$ by $[q, v(\omega, \omega, 1, \omega)]$

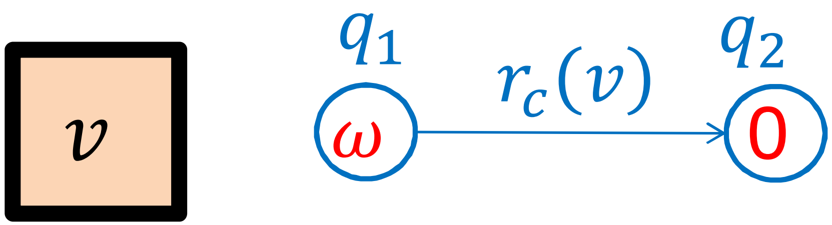
Lock-free Shared Memory

Fact 1: Every ω-configuration of the graph contains only 0´s and ω´s.



Lock-free Shared Memory

Fact 1: Every ω-configuration of the graph contains only 0´s and ω´s.



$$[\mathrm{q}, \mathrm{v}, (\omega, 0, \dots)] \rightarrow [\mathrm{q}, \mathrm{v}, (\omega, 1, \dots)]$$
$$\rightarrow [\mathrm{q}, \mathrm{v}, (\omega, 2, \dots)]$$
$$\rightarrow [\mathrm{q}, \mathrm{v}, (\omega, 3, \dots)] \cdots$$

Replace $[\mathrm{q}, \mathrm{v}, (\omega, 1, \dots)]$ by $[\mathrm{q}, \mathrm{v}(\omega, \omega, \dots)]$

Lock-free Shared Memory

Fact 2: In every run, the ω's grow monotonically.

Lock-free Shared Memory

Fact 2: In every run, the ω's grow monotonically.

Consequence: Every simple path of the Karp-Miller graph has length $n_l \cdot n_v \cdot n_c$ where

- n_l number of states of leader template
- n_v number of values
- n_c number of states of contributor template

Therefore: coverability is in NP.

Lock-free Shared Memory

Compare with: The coverability problem for a fixed number of contributor processes is PSPACE-complete.

Shared memory without locking

Theorem (E.,Ganty, Majumdar 2013)

The problem remains NP-complete if the template is a polytime Turing machine

This means we cannot distribute an exponentially long computation onto exponentially many machines so that each machine only does polynomial work.

Not covered work and open questions

Termination

Not covered work and open questions

Termination

Temporal logics

Not covered work and open questions

Termination

Temporal logics

Implementations

Not covered work and open questions

And if the processes know N?

Termination

Temporal logics

Implementations


That´s
all!